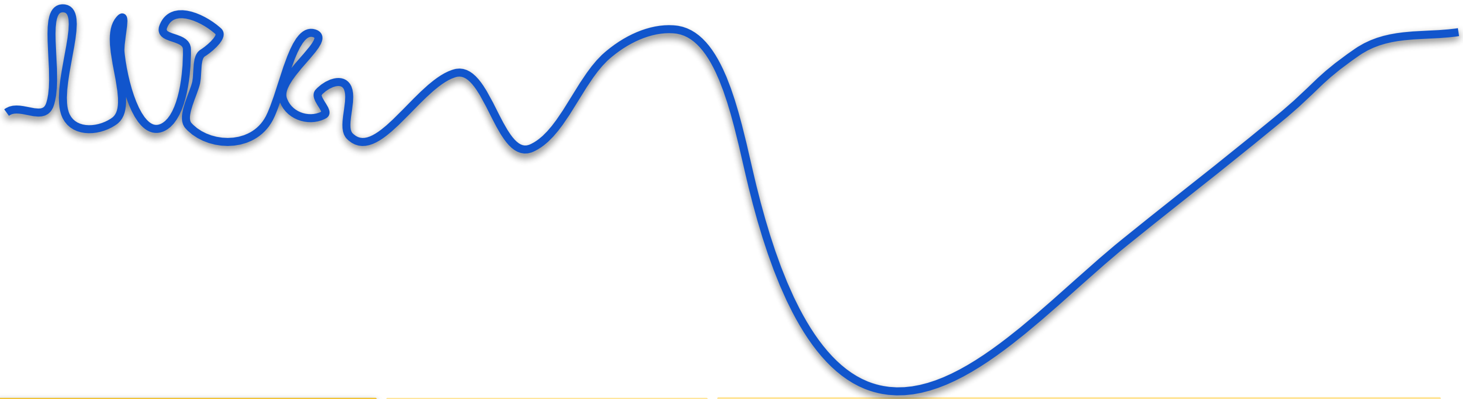
Computing with Signals



DA 623

Jan - May 2023

IIT Guwahati

Instructors: Neeraj Sharma

Lecture-04

Signal

Model

or

Representation

Processing

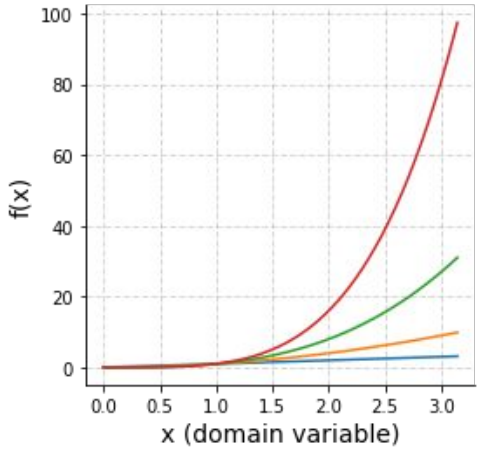
Signal ⇒ Model

or

Representation

- Polynomial representation

Focus of this lecture



Model

Taylor Series:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

Model

Taylor Series:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

Examples:

$$\sin(x) \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

Model

Taylor Series:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

Examples:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

Model

Taylor Series:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

Examples:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

More examples:

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \quad \text{for all } x$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots \quad \text{for all } x$$

$$\tan x = \sum_{n=1}^{\infty} \frac{B_{2n} (-4)^n (1-4^n)}{(2n)!} x^{2n-1} = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots \quad \text{for } |x| < \frac{\pi}{2}$$

$$\sec x = \sum_{n=0}^{\infty} \frac{(-1)^n E_{2n}}{(2n)!} x^{2n} = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \cdots \quad \text{for } |x| < \frac{\pi}{2}$$

$$\arcsin x = \sum_{n=0}^{\infty} \frac{(2n)!}{4^n (n!)^2 (2n+1)} x^{2n+1} = x + \frac{x^3}{6} + \frac{3x^5}{40} + \cdots \quad \text{for } |x| \le 1$$

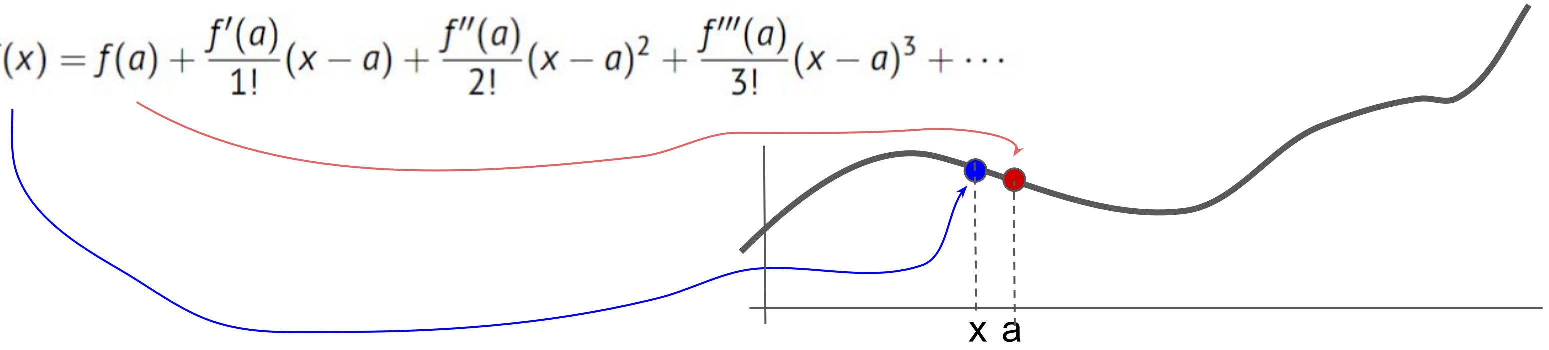
Limitations

Taylor Series:

- The function is differentiable
- Works like a charm if you know the function (in closed form) apriori
- Approximation only in the neighborhood of the sampled point

Using in practice requires derivative information of the signal.

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$



Signal ⇒ Model *or* Representation ⇒ Processing

Model

Polynomial Series:

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

Model

Polynomial Series:

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

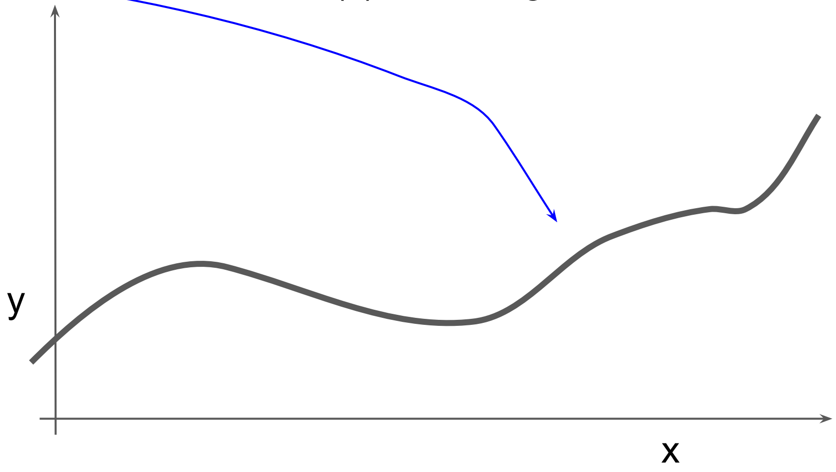
A univariate polynomial of degree n with real or complex coefficients has n complex roots, if counted with their multiplicities.

Model

Polynomial Series:

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \cdots$$

Fit f(x) to this signal?



Model

Polynomial Series:

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3$$

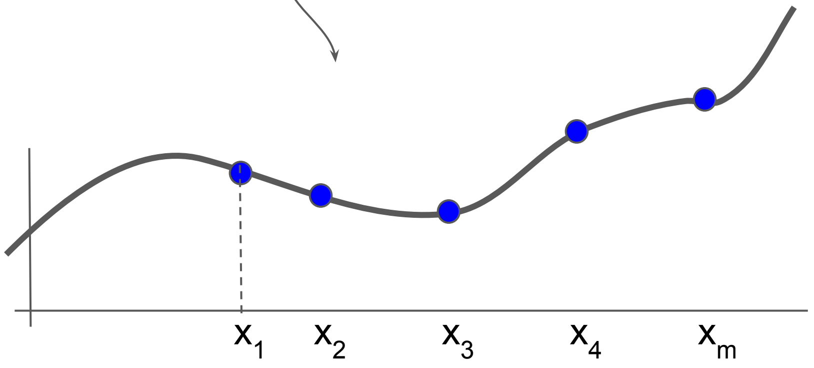
Step 1: Sample the signal at at least (P+ 1) points (why P+1)? (P is the degree)

$$y_1 = a_0 + a_1x_1 + a_2x_1^2 + a_3x_3^3$$

$$y_2 = a_0 + a_1x_2 + a_2x_2^2 + a_3x_2^3$$

$$\vdots$$

$$y_m = a_0 + a_1x_m + a_2x_m^2 + a_3x_m^3$$



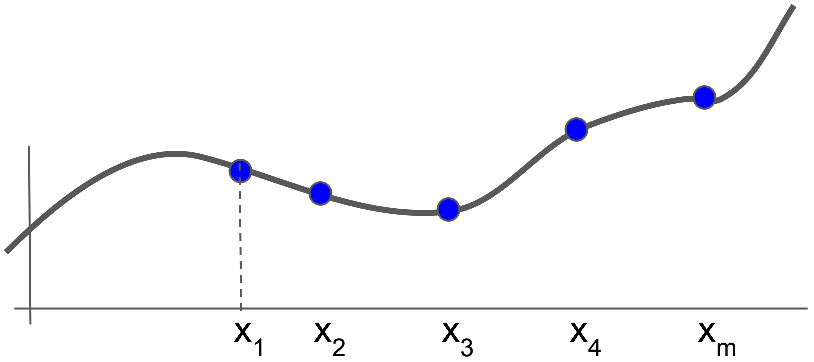
Model

Polynomial Series:

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 \cdot$$

Step 1: Sample the signal at at least (n) points (why?). [n-1 is the degree of f(x)]

$$\begin{pmatrix} y_1 \\ y_2 \\ \\ \\ y_m \end{pmatrix} = \begin{bmatrix} 1 & x_1 & x_1^2 & \dots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \dots & x_2^{n-1} \\ 1 & x_3 & x_3^2 & \dots & x_3^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_m & x_m^2 & \dots & x_m^{n-1} \end{bmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \\ \\ a_{n-1} \end{pmatrix}$$



Model

Polynomial Series:

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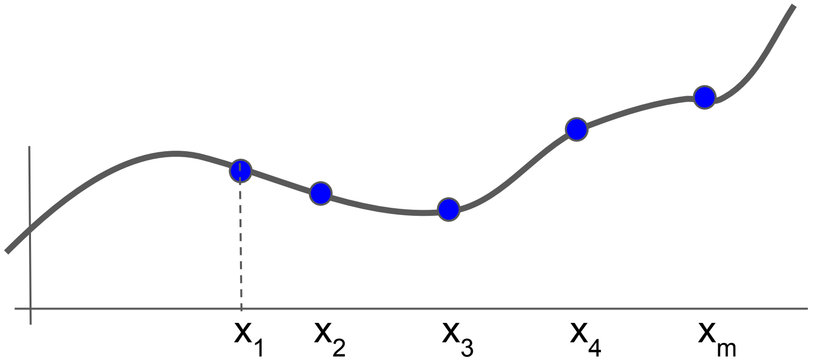
Step 2: Solve y = Xa



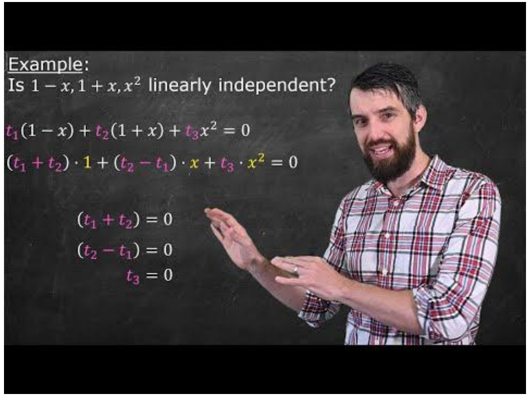
Summary, Polynomial series approximation,

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

- Can model (or represent) any polynomial of degree n
- Parameters of the model are $\{a_o, a_1, \ldots, a_n\}$
- Estimating the parameters requires a regression approach

Summary, Polynomial series approximation,

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$



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