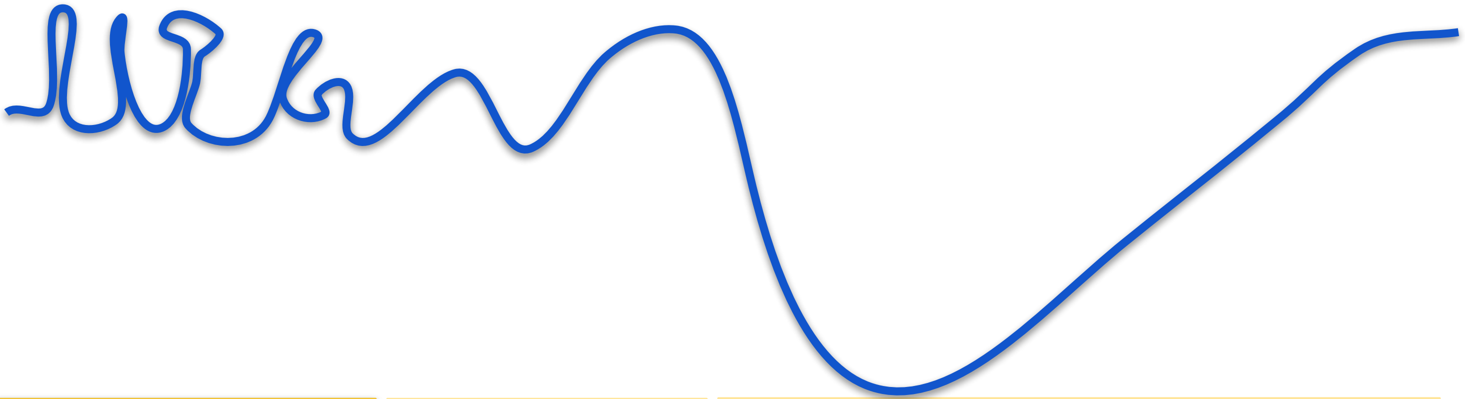
Computing with Signals



DA 623

Jan - May 2023

IIT Guwahati

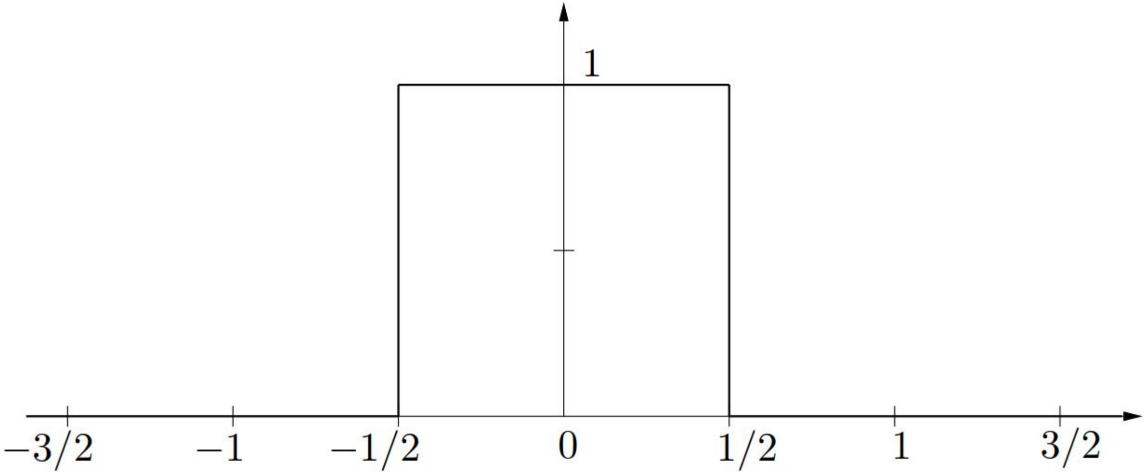
Instructors: Neeraj Sharma

Lecture-07

- Polynomial series representation
- Fourier series representation

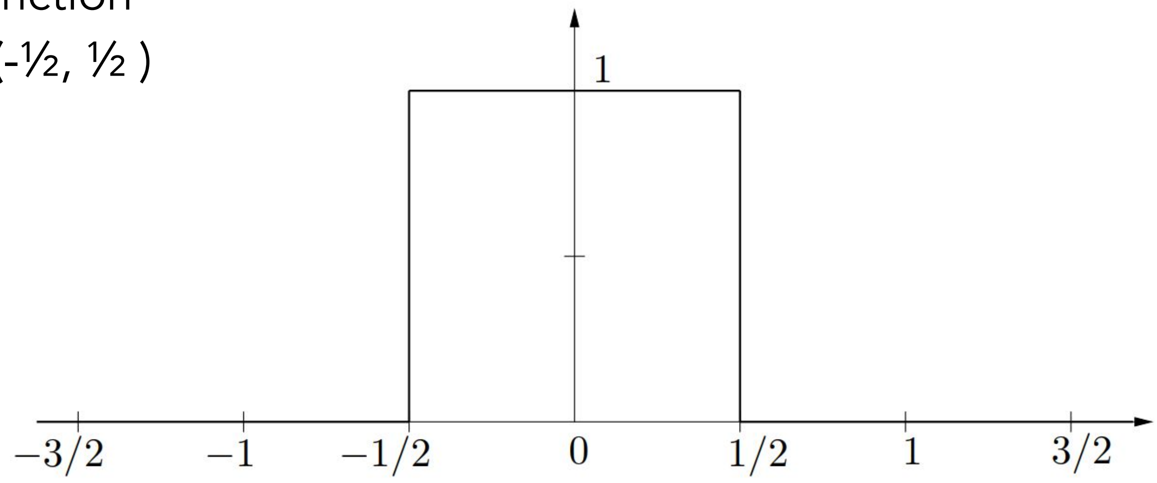
- Polynomial series representation
- Fourier series representation
- Fourier transform representation

$$\Pi(t) = \begin{cases} 1 & |t| < 1/2 \\ 0 & |t| \geq 1/2 \end{cases}$$



$$\Pi(t) = \begin{cases} 1 & |t| < 1/2 \\ 0 & |t| \geq 1/2 \end{cases}$$

Top hat function, indicator function, or characteristic function for the interval (-½, ½)

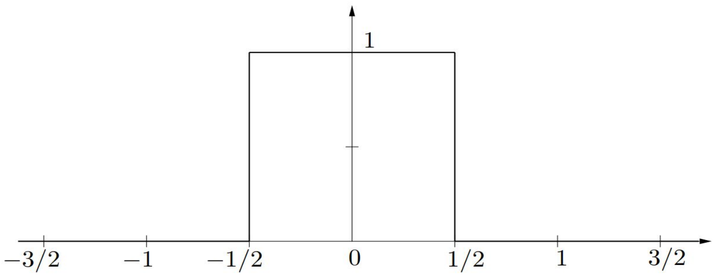


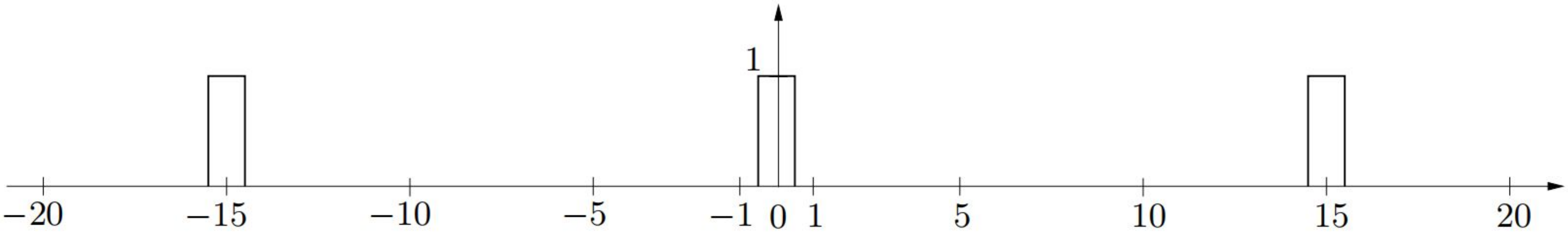
- rect(t) is not periodic

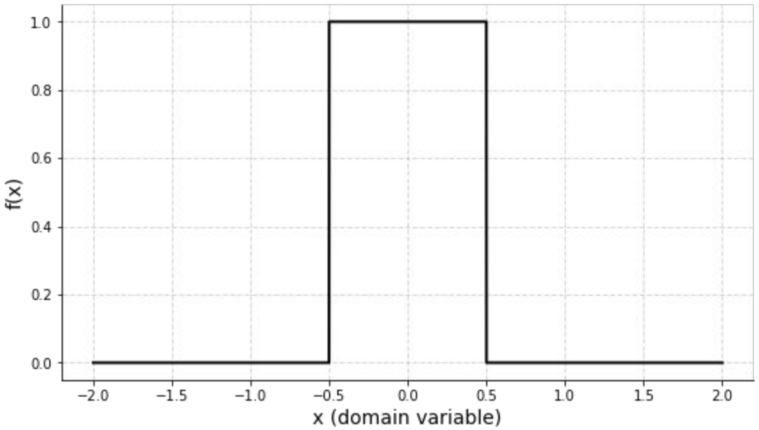
- It does not have Fourier series

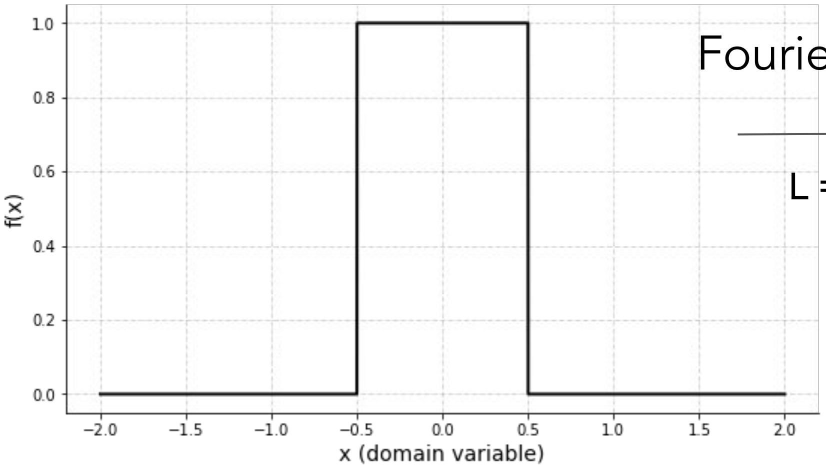
- Let's create another function by periodically repeating rect(t) with a long period

$$\Pi(t) = \begin{cases} 1 & |t| < 1/2 \\ 0 & |t| \geq 1/2 \end{cases}$$



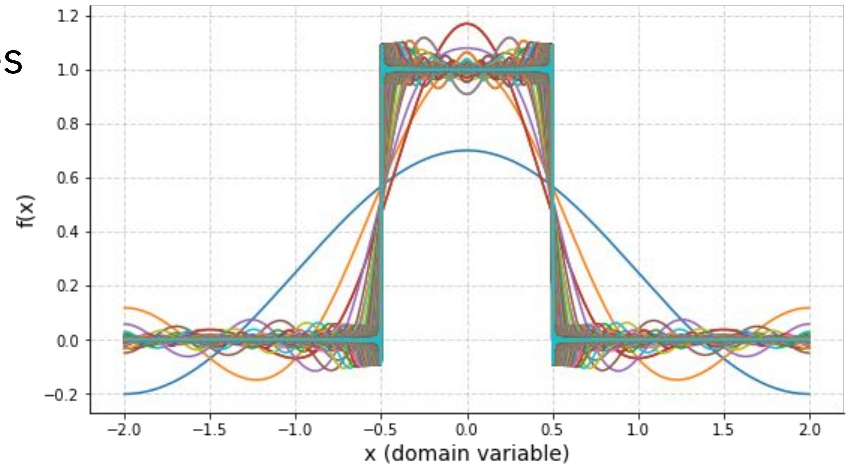



1.0
0.8
0.6
0.4
0.2
0.0
f(x)
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
x (domain variable)



Fourier Series

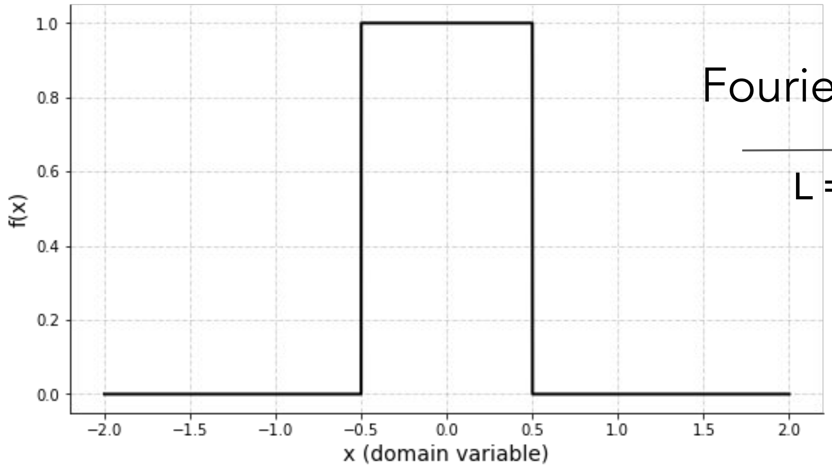
L = 4



Colors indicate approximation with increasing m

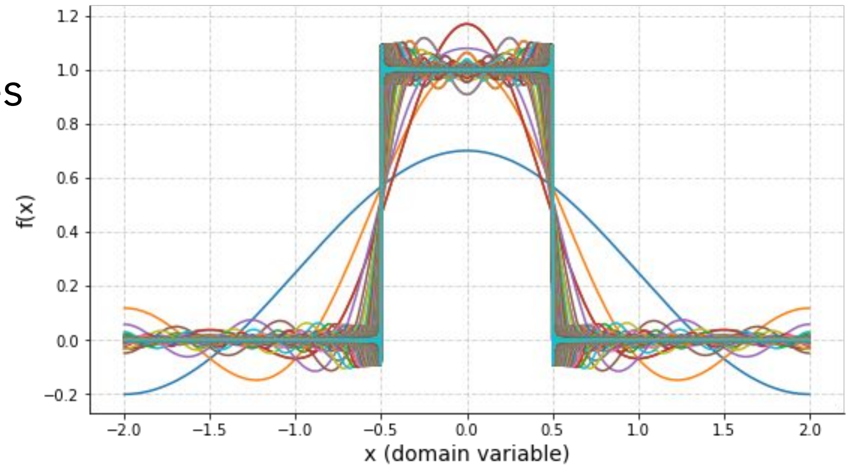
$$f(x) = \frac{a_0}{2} + \sum_{m=1}^{M} \left(a_m \cos(\frac{m\pi x}{L}) + b_m \sin(\frac{m\pi x}{L}) \right)$$

$$\begin{aligned} a_0 &= \frac{1}{L}\int_{-L}^{L} f(x)dx \\ a_m &= \frac{1}{L}\int_{-L}^{L} f(x)\cos(\frac{m\pi x}{L})dx, \text{ and} \\ b_m &= \frac{1}{L}\int_{-L}^{L} f(x)\sin(\frac{m\pi x}{L})dx. \end{aligned}$$



Fourier Series

L = 2



Colors indicate approximation with increasing m

$$f(x) = \frac{a_0}{2} + \sum_{m=1}^{M} \left(a_m \cos(\frac{m\pi x}{L}) + b_m \sin(\frac{m\pi x}{L}) \right)$$

$$a_0 = \frac{1}{L} \int_{-L}^{L} f(x) dx$$

$$a_m = \frac{1}{L} \int_{-L}^{L} f(x) \cos(\frac{m\pi x}{L}) dx, \text{ and}$$

$$b_m = \frac{1}{L} \int_{-L}^{L} f(x) \sin(\frac{m\pi x}{L}) dx.$$

Fourier Series

Exponential representation

$$f(x) = \sum_{k=-\infty}^{\infty} c_k e^{j\pi m x/L}$$

$$c_0 = a_0/2$$

$$c_m = \frac{(a_m - jb_m)}{2}, \text{ for m>0}$$

$$c_{-m} = \frac{(a_m + jb_m)}{2}, \text{ for m<0}$$

$$f(x) = \sum_{k=-\infty}^{\infty} c_k e^{j\pi m x/L}$$

$$c_0 = a_0/2$$

$$c_m = \frac{(a_m - jb_m)}{2}, \text{ for m>0}$$

$$c_{-m} = \frac{(a_m + jb_m)}{2}, \text{ for m<0}$$

Equivalently,

$$f(x) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi kx}{T}}$$

$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} e^{-j\frac{2\pi kx}{T}} f(x)dx$$

$$f(x) = \sum_{k=-\infty}^{\infty} c_k e^{j\pi mx/L}$$

$$c_0 = a_0/2$$

$$c_m = \frac{(a_m - jb_m)}{2}, \text{ for m>0}$$

$$c_{-m} = \frac{(a_m + jb_m)}{2}, \text{ for m<0}$$

Equivalently,

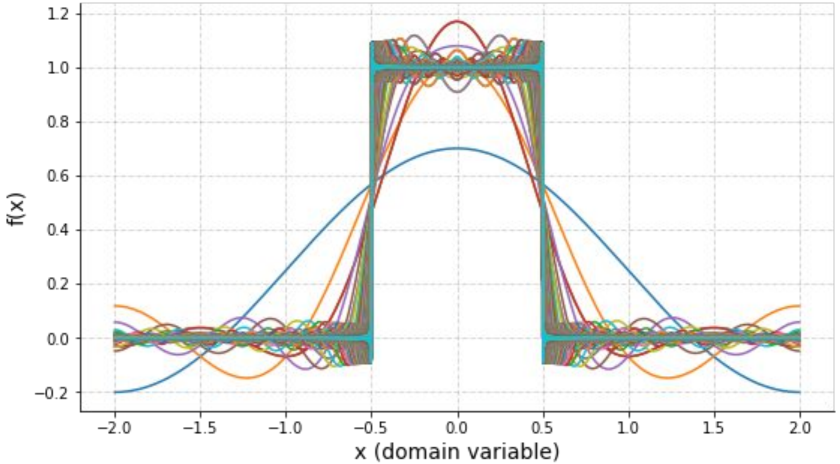
$$f(x) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi kx}{T}}$$

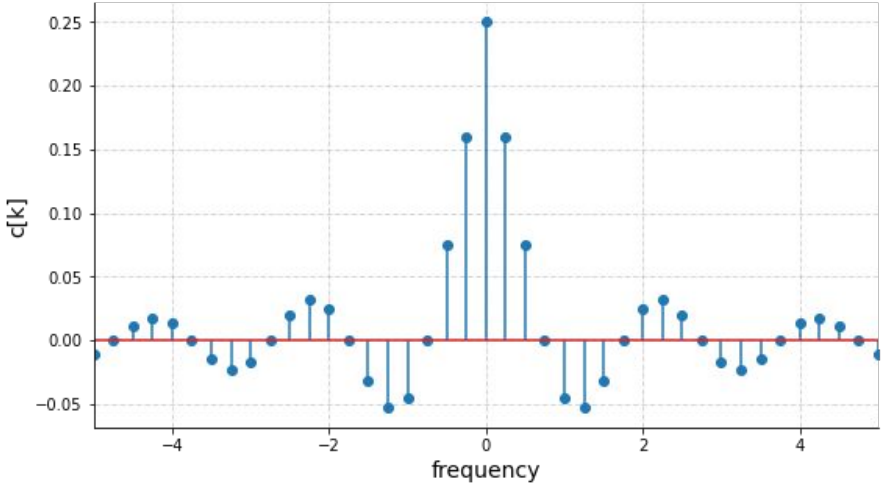
$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} e^{-j\frac{2\pi kx}{T}} f(x)dx$$

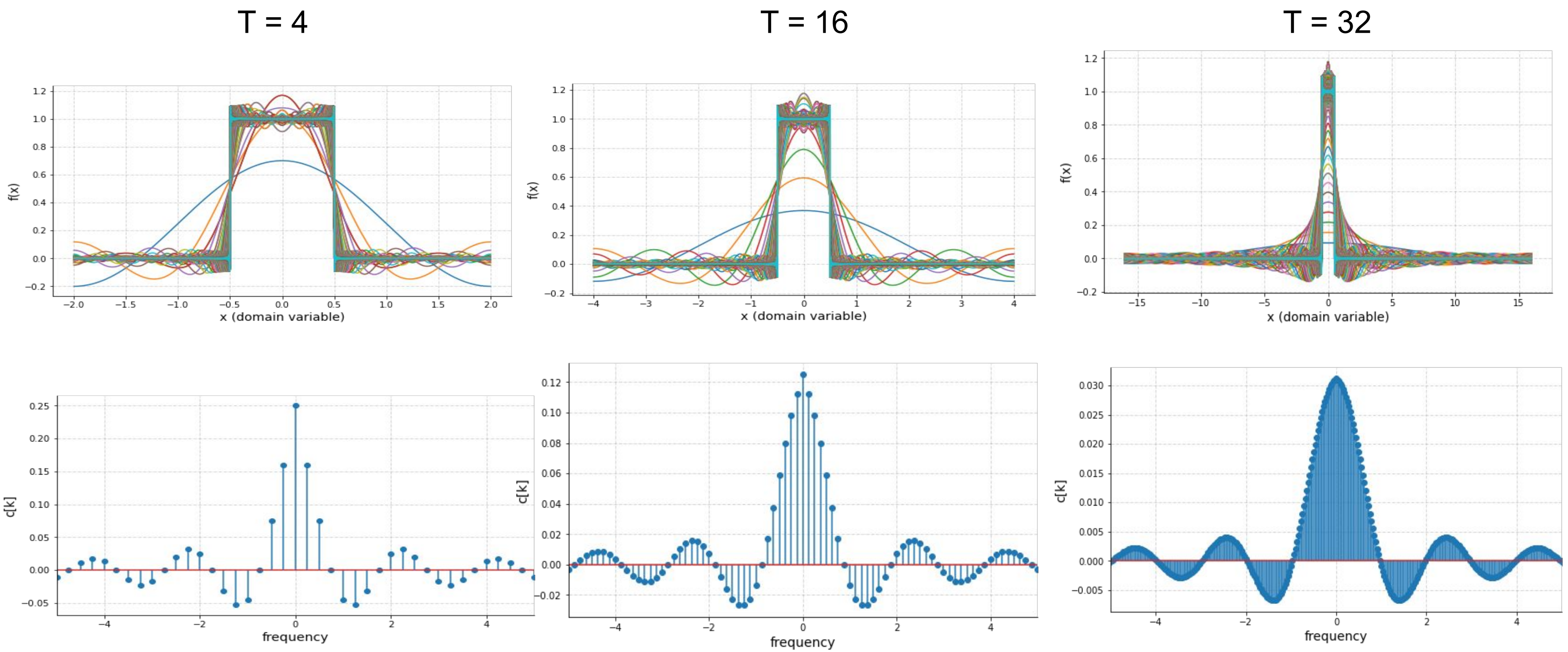
Notion of frequency: $\frac{k}{T}$

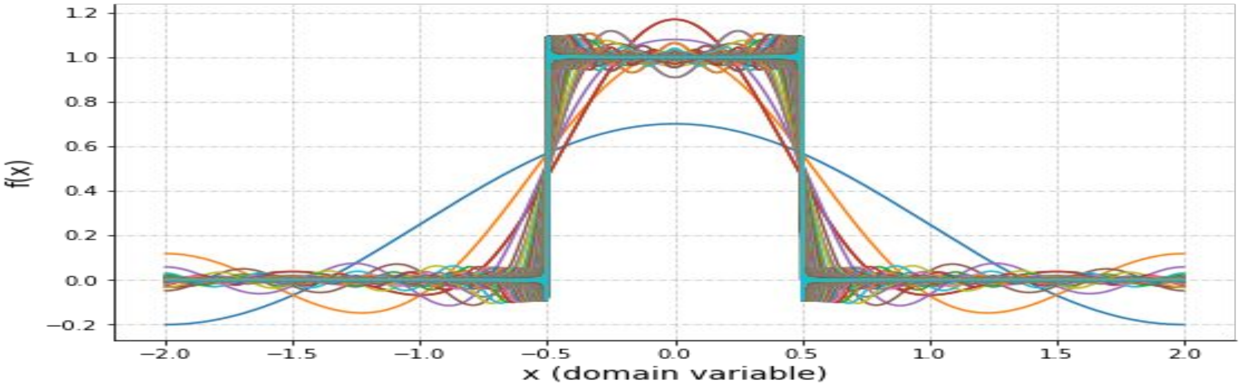
$$f(x) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi kx}{T}}$$

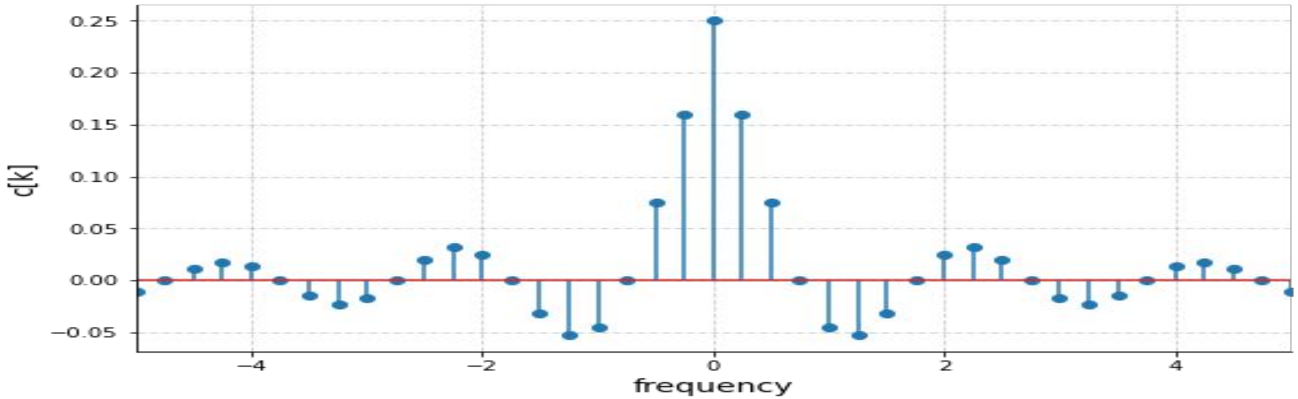
$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} e^{-j\frac{2\pi kx}{T}} f(x)dx$$






T = 4
T = 16
T = 32
f(x)
x (domain variable)
c[k]
frequency


f(x)
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
x (domain variable)


c[k]
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
−4
−2
0
2
4
frequency

Thank you