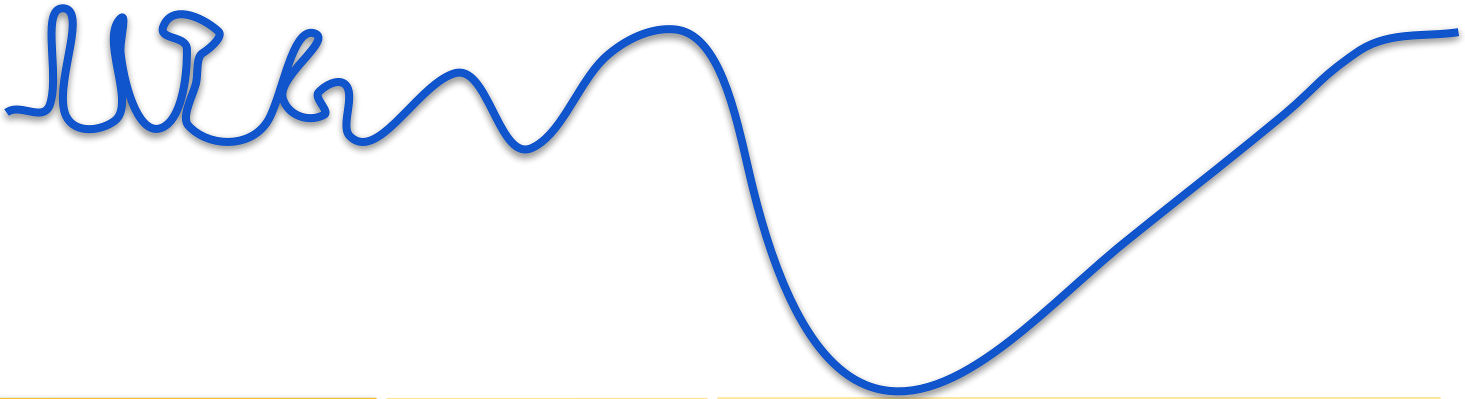
Computing with Signals



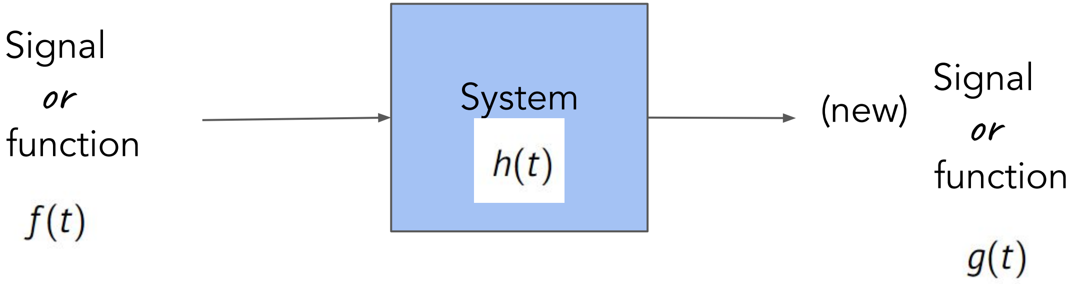
DA 623

Jan - May 2023

IIT Guwahati

Instructors: Neeraj Sharma

Lecture-13-[09-Feb]



$$g(t) = h(t) * f(t)$$

$$g(t) = \int_{-\infty}^{\infty} h(t-u) f(u) du$$

Sampling and Interpolation

$$g(x,t) = \frac{1}{\sqrt{2\pi t}} e^{\frac{-x^2}{2t}}, \; t > 0$$

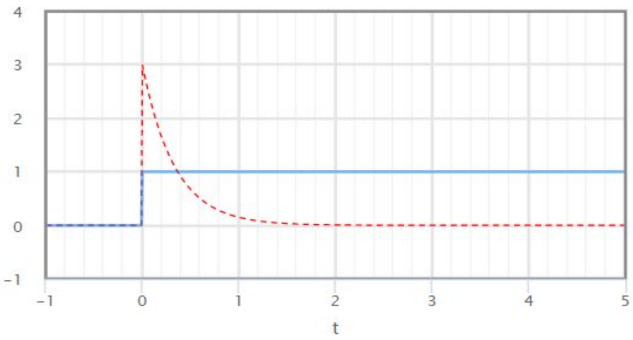
$$\rho_p(x) = \sum_{k=-\infty}^{\infty} \rho(x - kp)$$

$$\rho_p(x) = \sum_{k=-\infty}^{\infty} \rho(x - pk) = \sum_{k=-\infty}^{\infty} \delta(x - kp) * \rho(x)$$

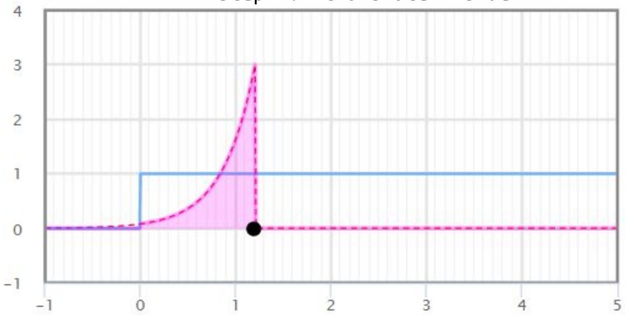
$$= \left(\sum_{k=-\infty}^{\infty} \delta(x - kp) \right) * \rho(x)$$

Example: `https://lpsa.swarthmore.edu/Convolution/CI.html`

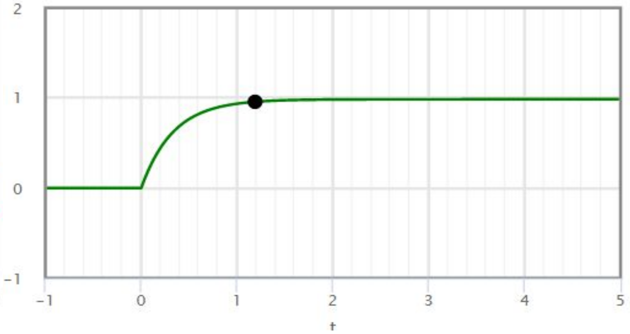
Signals



Step 1: Flip one of them,
Step 2: translate inside



Step 3: Product and Integrate,
Step 4: Go to step 2



Periodizing a function $$\rho_p(x) = \sum_{k=-\infty}^{\infty} \rho(x - kp)$$

- Shah function
- Comb function
- Train of Diracs

$$\text{Ш}_p(x) = \sum_{k=-\infty}^{\infty} \delta(x - kp)$$

$$\rho_p = \text{Ш}_p * \rho \,.$$

$$\text{Ш}_p(x) = \sum_{k=-\infty}^{\infty} \delta(x - kp) \quad \text{or} \quad \text{Ш}_p = \sum_{k=-\infty}^{\infty} \delta_{kp}$$

$$\text{Ш}_p(x) = \sum_{k=-\infty}^{\infty} \delta(x - kp) \quad \text{or} \quad \text{Ш}_p = \sum_{k=-\infty}^{\infty} \delta_{kp}$$

$$\langle \text{Ш}_p, \varphi \rangle = \left\langle \sum_{k=-\infty}^{\infty} \delta_{kp}, \varphi \right\rangle = \sum_{k=-\infty}^{\infty} \langle \delta_{kp}, \varphi \rangle = \sum_{k=-\infty}^{\infty} \varphi(kp)$$

The Shah function provides one way to periodizing a function

$$(f * \text{III}_p)(t) = \sum_{k=-\infty}^{\infty} f(t - pk)$$

Of special interest when f is zero for $|t| \geq p/2$ as then,

$$\Pi_p f = f$$

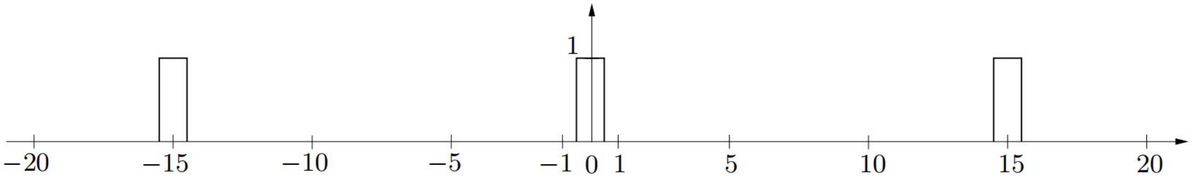
$$f = \Pi_p(f * \text{III}_p)$$

The Shah function provides one way to periodizing a function

$$(f * \text{III}_p)(t) = \sum_{k=-\infty}^{\infty} f(t - pk)$$

Can you recall we used this approach in earlier classes?

$$\Pi(t) = \begin{cases} 1 & |t| < 1/2 \\ 0 & |t| \geq 1/2 \end{cases}$$



The Shah function provides one way to periodizing a function

$$(f * \text{Ш}_p)(t) = \sum_{k=-\infty}^{\infty} f(t - pk)$$

The Shah function also provides one way sampling a function

$$f(x)\text{Ш}(x) = \sum_{k=-\infty}^{\infty} f(x)\delta(x-k) = \sum_{k=-\infty}^{\infty} f(k)\delta(x-k)$$

The Shah function also provides one way sampling a function

$$f(x)\text{Ш}(x) = \sum_{k=-\infty}^{\infty} f(x)\delta(x-k) = \sum_{k=-\infty}^{\infty} f(k)\delta(x-k)$$

Sampling at arbitrary but regularly spaced points

$$f(x)\text{Ш}_p(x) = \sum_{k=-\infty}^{\infty} f(kp)\delta(x-kp)$$

Scaling the Shah function

$$\text{Ш}_p(x) = \sum_{k=-\infty}^{\infty} \delta(x - kp)$$

$$\text{Ш}(px) = \sum_{k=-\infty}^{\infty} \delta(px - k)$$

Fourier Transform of Shah function

$$\text{III}(x) = \sum_{k=-\infty}^{\infty} \delta(x-k)$$

$$\mathcal{F}\text{III}(s) = \sum_{k=-\infty}^{\infty} e^{-2\pi iks} = \sum_{k=-\infty}^{\infty} e^{2\pi iks} = \text{III}$$

Fourier Transform of Shah function

$$\text{III}(x) = \sum_{k=-\infty}^{\infty} \delta(x-k)$$

$$\mathcal{F}\text{III}(s) = \sum_{k=-\infty}^{\infty} e^{-2\pi iks} = \sum_{k=-\infty}^{\infty} e^{2\pi iks} = \text{III}$$

$$\sum_{n=-N}^{N} e^{2\pi int}$$

$$\text{Ш}(x) = \sum_{k=-\infty}^{\infty} \delta(x-k)$$

$$\begin{aligned}
\mathcal{F}\text{Ш}_p(s) &= \frac{1}{p}\mathcal{F}\left(\text{Ш}\left(\frac{x}{p}\right)\right) \\
&= \frac{1}{p} p \mathcal{F}\text{Ш}(ps) \quad \text{(stretch theorem)} \\
&= \text{Ш}(ps) \\
&= \frac{1}{p}\text{Ш}_{1/p}(s)
\end{aligned}$$

$$\mathcal{F}\text{Ш}(s) = \sum_{k=-\infty}^{\infty} e^{-2\pi i k s} = \sum_{k=-\infty}^{\infty} e^{2\pi i k s} = \text{Ш}$$

Thank you!