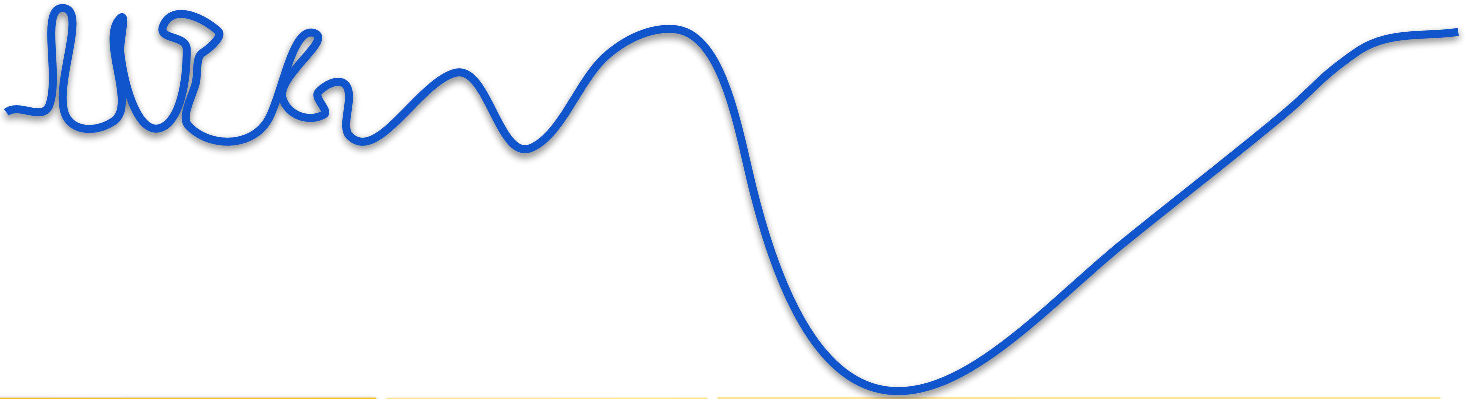
Computing with Signals



DA 623

Jan - May 2023

IIT Guwahati

Instructors: Neeraj Sharma

Lecture-14-[10-Feb]

Sampling and Interpolation

Recap

The Shah function provides one way to periodizing a function

$$(f * \text{Ш}_p)(t) = \sum_{k=-\infty}^{\infty} f(t - pk)$$

The Shah function also provides one way sampling a function

$$f(x)\text{Ш}(x) = \sum_{k=-\infty}^{\infty} f(x)\delta(x-k) = \sum_{k=-\infty}^{\infty} f(k)\delta(x-k)$$

Clarification on action of delta(.)

"Distributions are what distributions do", in that fundamentally they are defined by how they act on "genuine" functions, those in S.

Discussion on board.

Review of the properties

Scaling the Shah function

$$\text{Ш}_p(x) = \sum_{k=-\infty}^{\infty} \delta(x - kp)$$

$$\text{Ш}(px) = \sum_{k=-\infty}^{\infty} \delta(px - k)$$

Fourier Transform of Shah function

$$\text{III}(x) = \sum_{k=-\infty}^{\infty} \delta(x-k)$$

$$\mathcal{F}\text{III}(s) = \sum_{k=-\infty}^{\infty} e^{-2\pi iks} = \sum_{k=-\infty}^{\infty} e^{2\pi iks} = \text{III}$$

Fourier Transform of Shah function

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$$\sum_{n=-N}^{N} e^{2\pi int}$$

$$\text{III}(x) = \sum_{k=-\infty}^{\infty} \delta(x-k)$$

$$\begin{aligned}
\mathcal{F}\text{III}_p(s) &= \frac{1}{p}\mathcal{F}\left(\text{III}\left(\frac{x}{p}\right)\right) \\
&= \frac{1}{p} p\mathcal{F}\text{III}(ps) \quad \text{(stretch theorem)} \\
&= \text{III}(ps) \\
&= \frac{1}{p}\text{III}_{1/p}(s)
\end{aligned}$$

$$\mathcal{F}\text{III}(s) = \sum_{k=-\infty}^{\infty} e^{-2\pi iks} = \sum_{k=-\infty}^{\infty} e^{2\pi iks} = \text{III}$$

Thank you