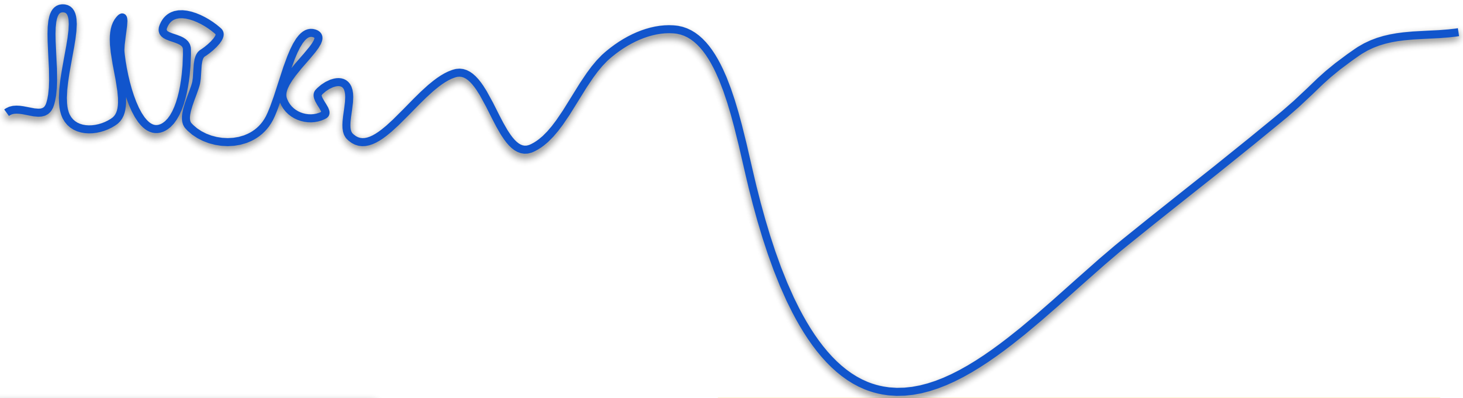
Computing with Signals



DA 623

Jan - May 2023 | IIT Guwahati

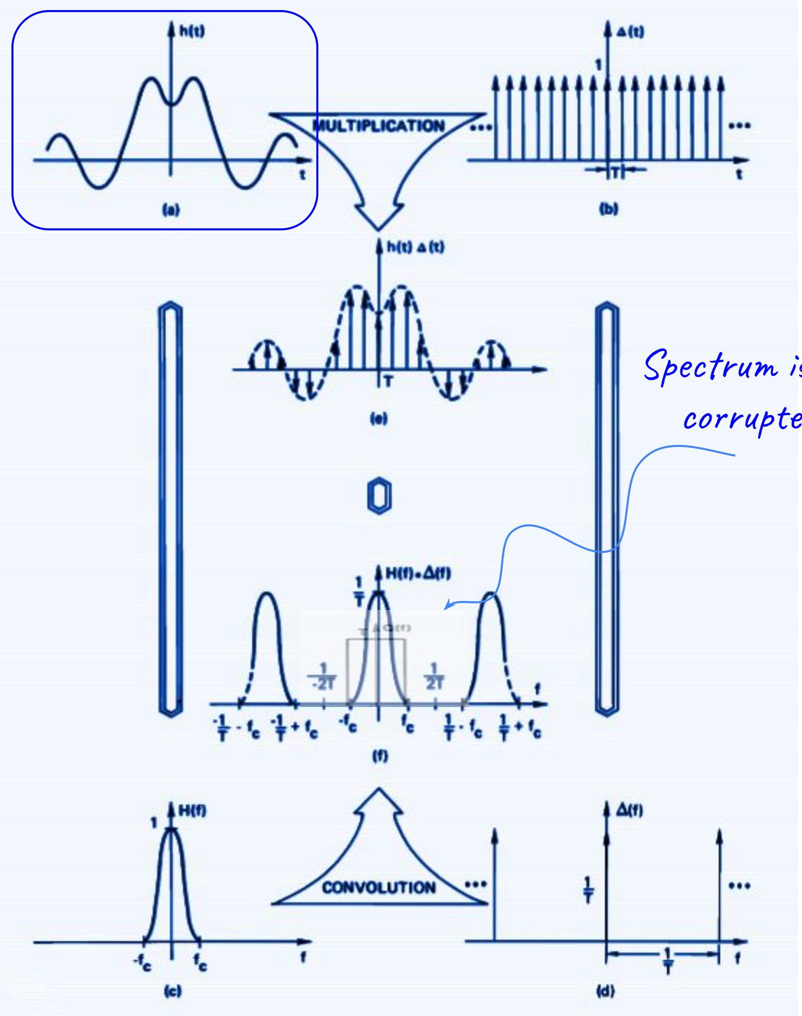
Instructors: Neeraj Sharma

Lecture-18-[22-Feb]

Sampling and Aliasing

through visualizations

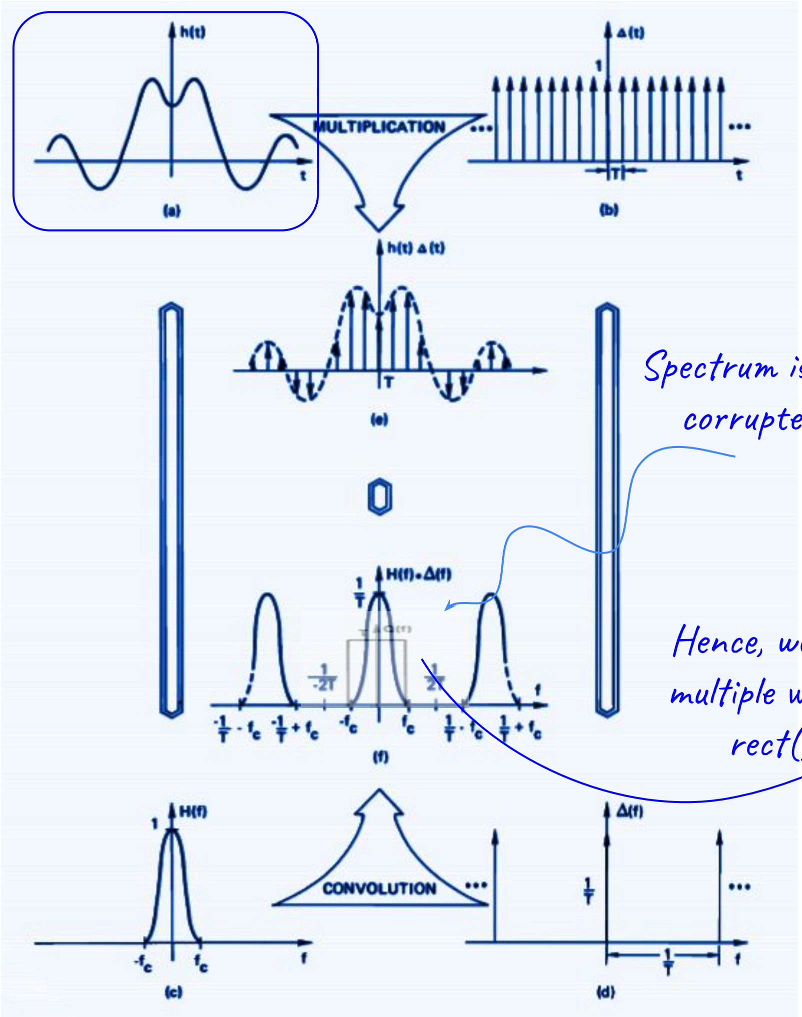
Acknowledgement: Multiple figures in this presentation are borrowed from the book "The Fast Fourier Transform and its Applications", by E. Oran Brigham. These figures are used here only for educational purpose.



Spectrum is not corrupted!

time-domain

frequency-domain

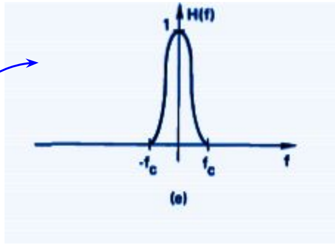


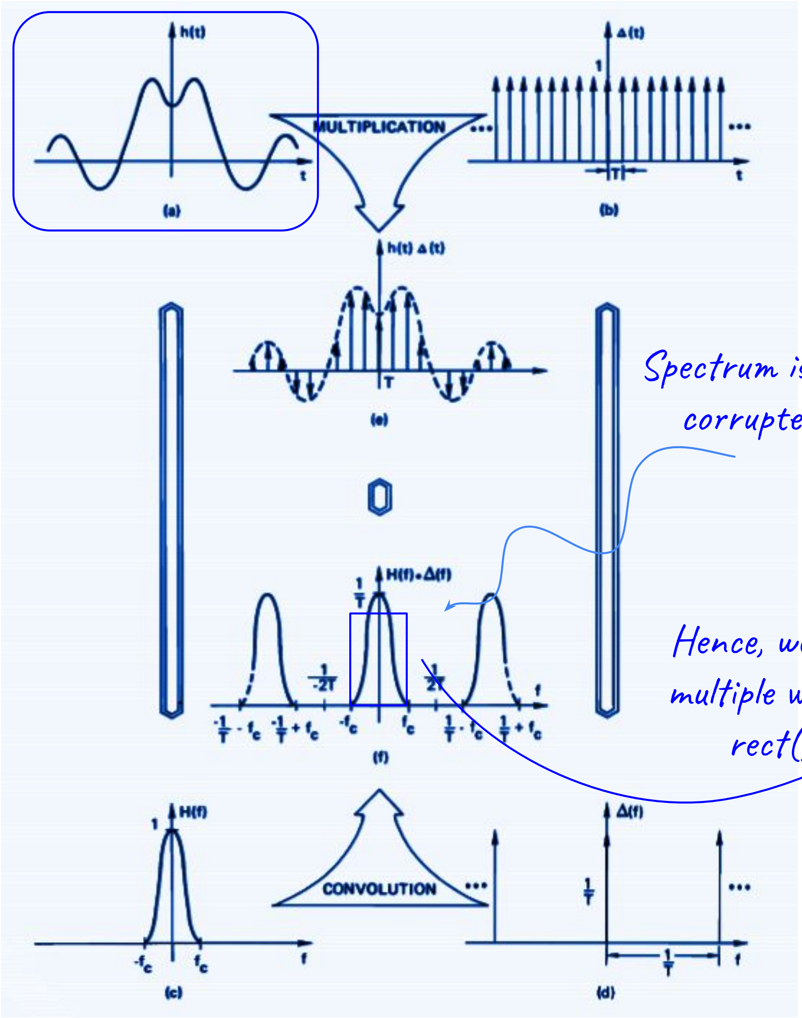
time-domain

frequency-domain

Spectrum is not corrupted!

Hence, we can multiple with a rect()



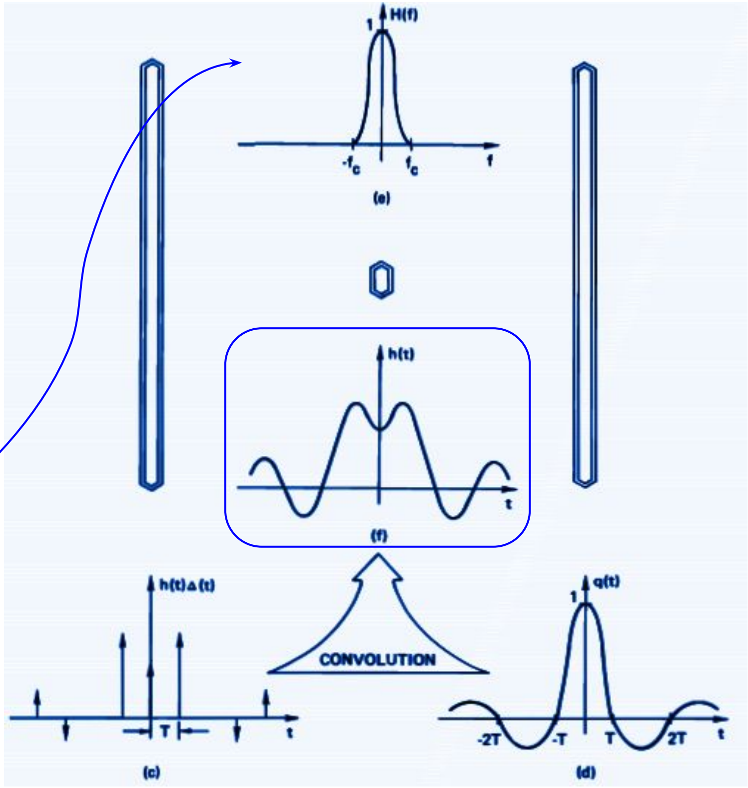


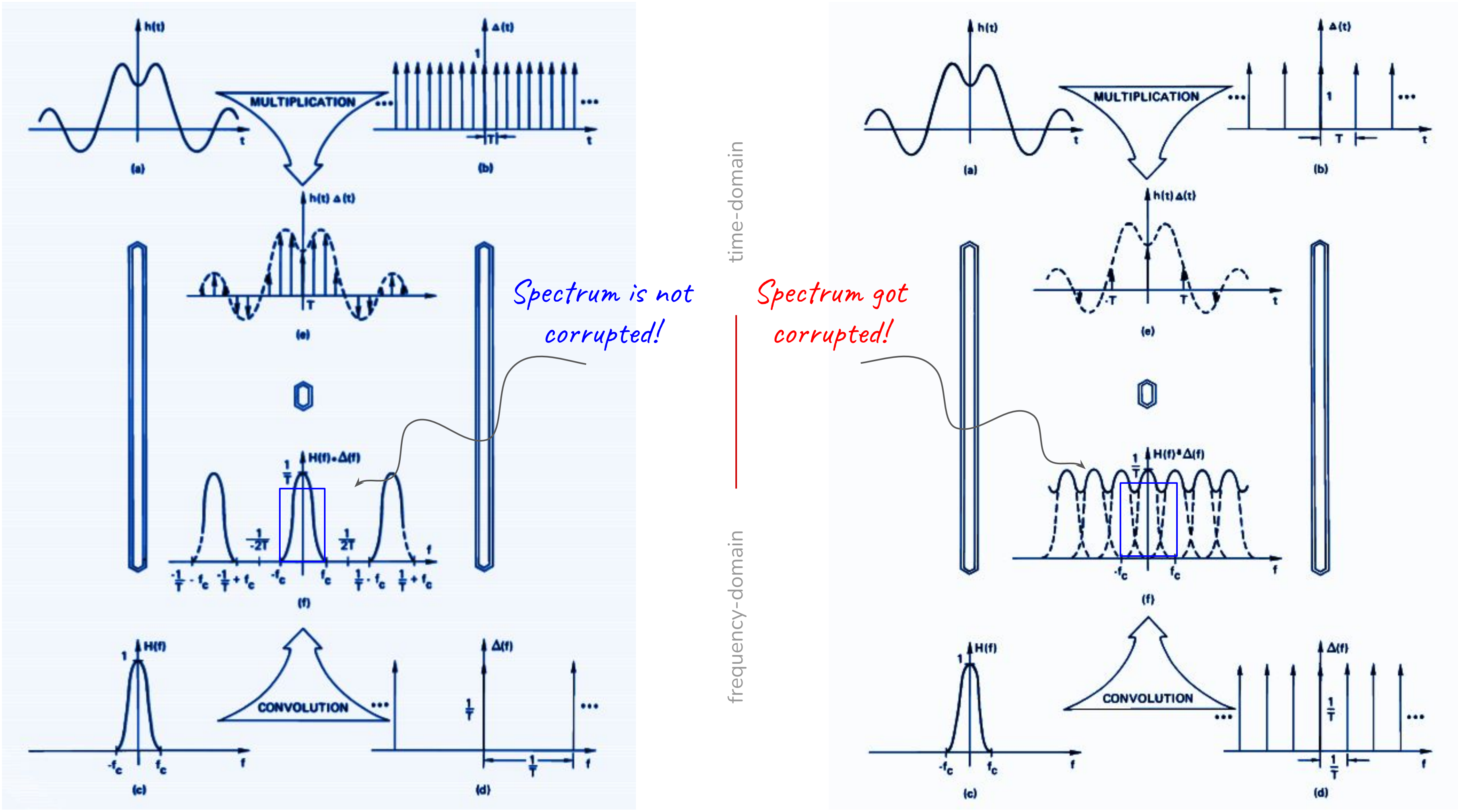
Spectrum is not corrupted!

Hence, we can multiple with a rect()

time-domain

frequency-domain

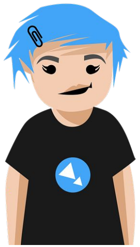




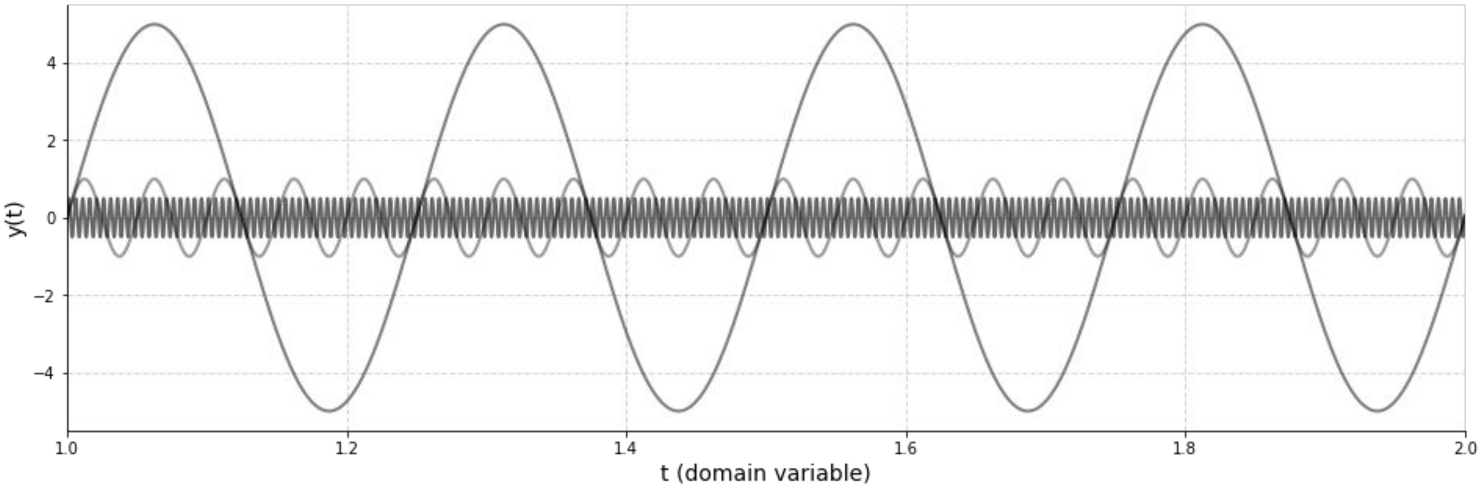
h(t)
Δ(t)
MULTIPLICATION
h(t) Δ(t)
H(f)∗Δ(f)
H(f)
Δ(f)
CONVOLUTION
(a)
(b)
(c)
(d)
(e)
(f)
time-domain
frequency-domain
Spectrum is not corrupted!
Spectrum got corrupted!

A Sampling Story

A Sampling Story

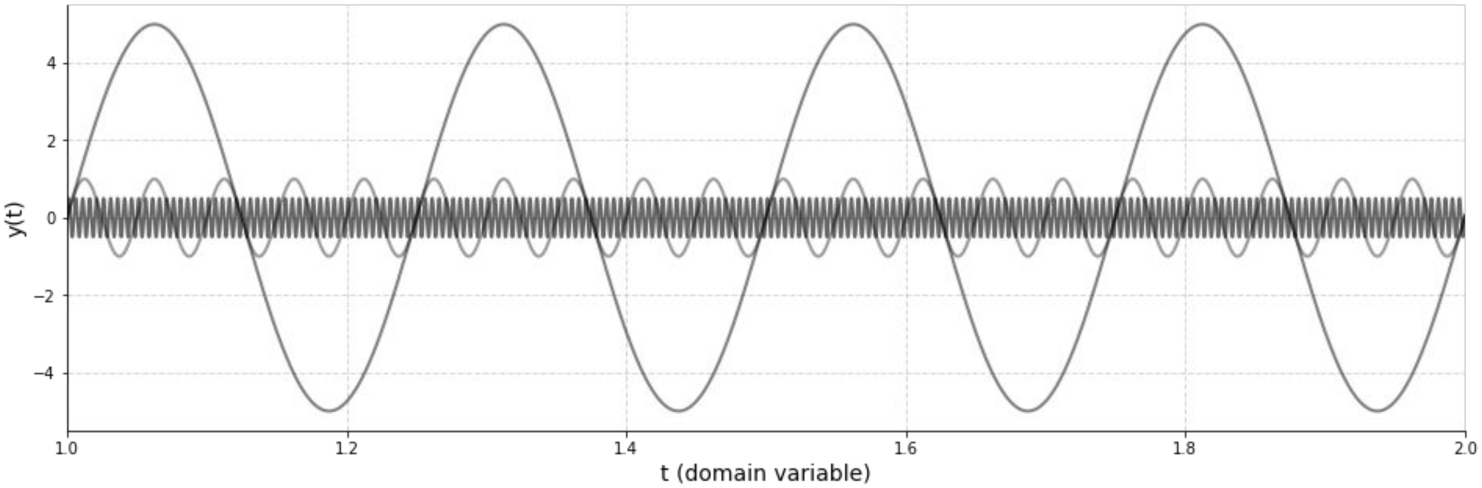


$y_1(t), y_2(t), y_3(t)$

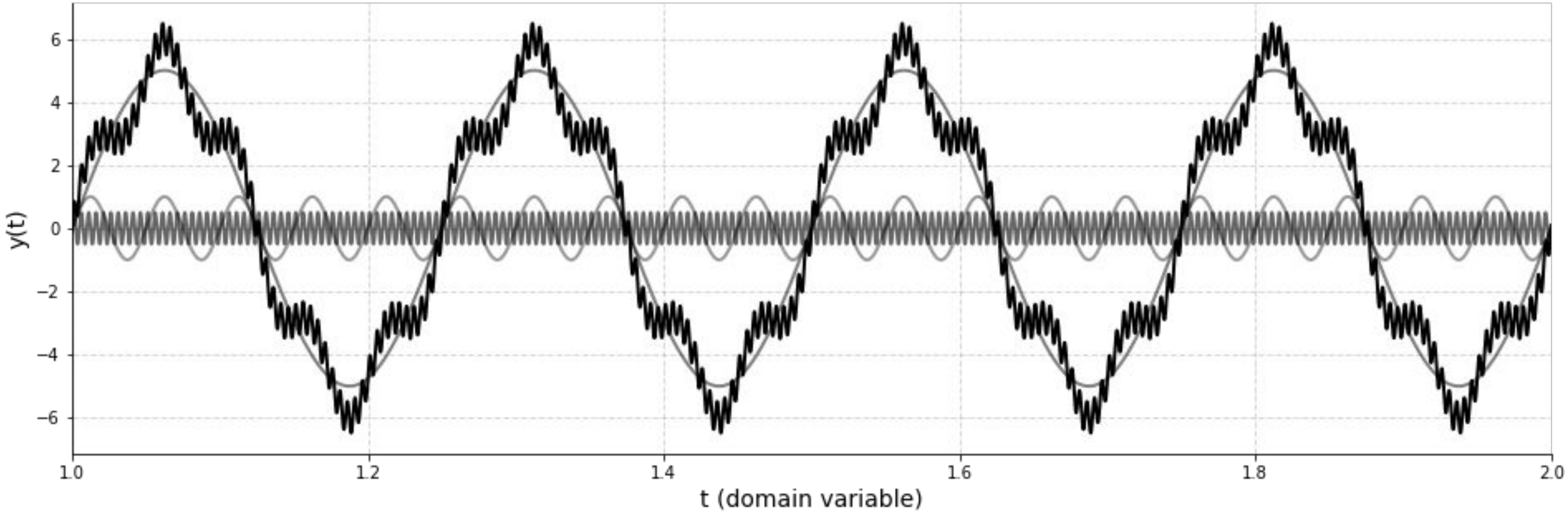


f_1= 4 Hz, f_2= 40 Hz, f_3=400 Hz

$y_1(t)$, $y_2(t)$, $y_3(t)$



$$y(t) = y_1(t) + y_2(t) + y_3(t)$$

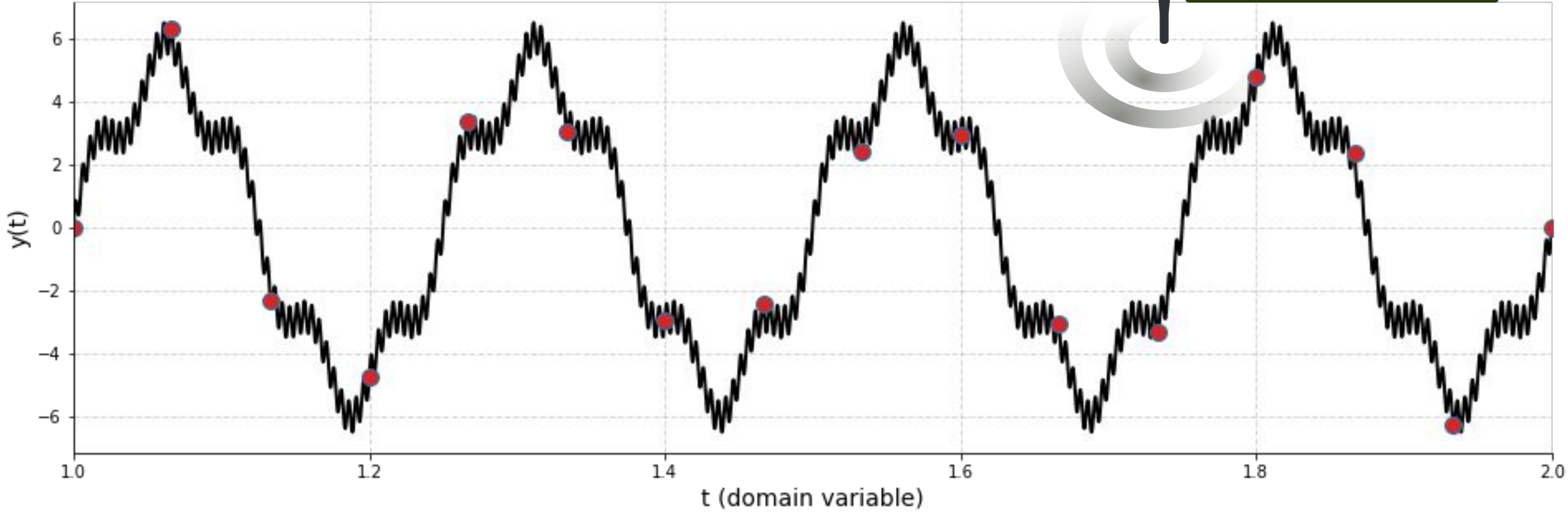



Odour sensor

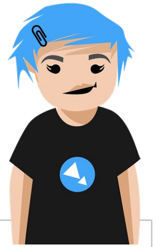
Assuming, T denotes the sampling period and n the sample index, we have:

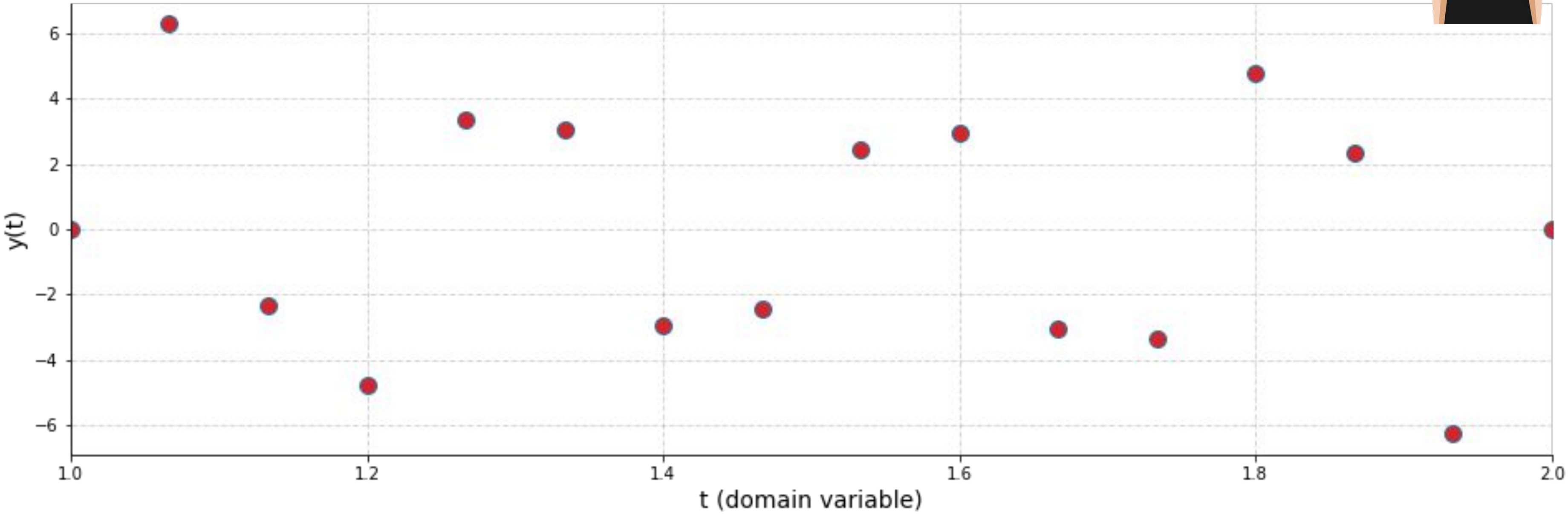
- Hardware parameter: $f_s = 15$ Hz, that makes, $T = 1/f_s$

$$y(nT) = y_1(nT) + y_2(nT) + y_3(nT)$$



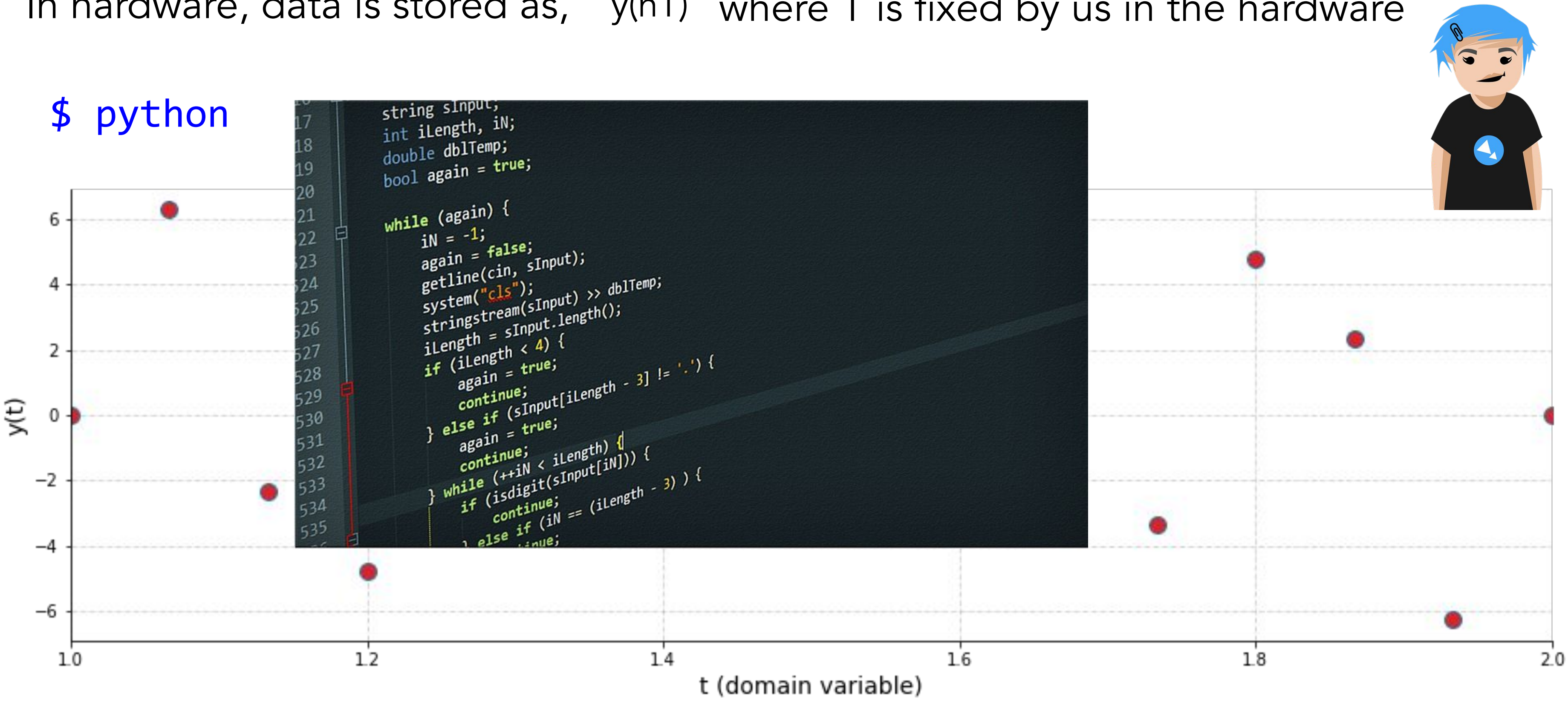
In hardware, data is stored as, $y(nT)$ where T is fixed by us in the hardware





In hardware, data is stored as, $y(nT)$ where T is fixed by us in the hardware

$ python

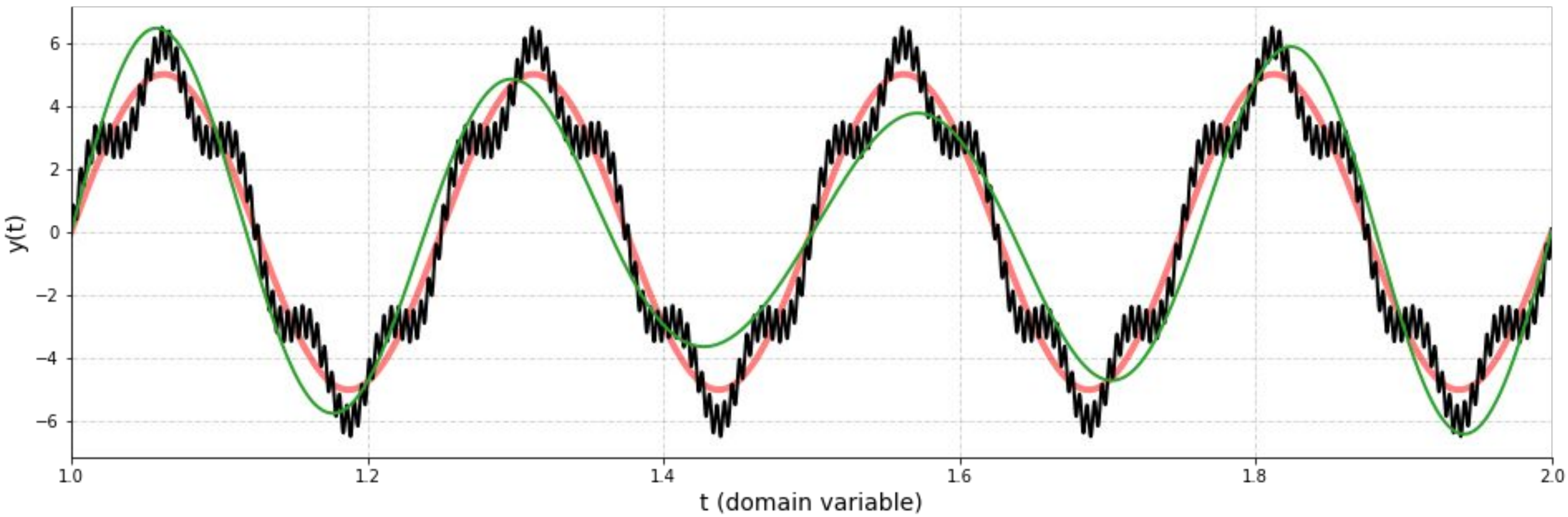


In hardware, data is stored as, $y(nT)$ where T is fixed by us in the hardware

$ python

$y_{hat}(t)$ obtained from $y(nT)$ via sinc interpolation

- Hardware parameter: f_s= 15 Hz, that makes, $T = 1/f_s$



1. Did it reconstruct $y(t)$?
2. If no, why?

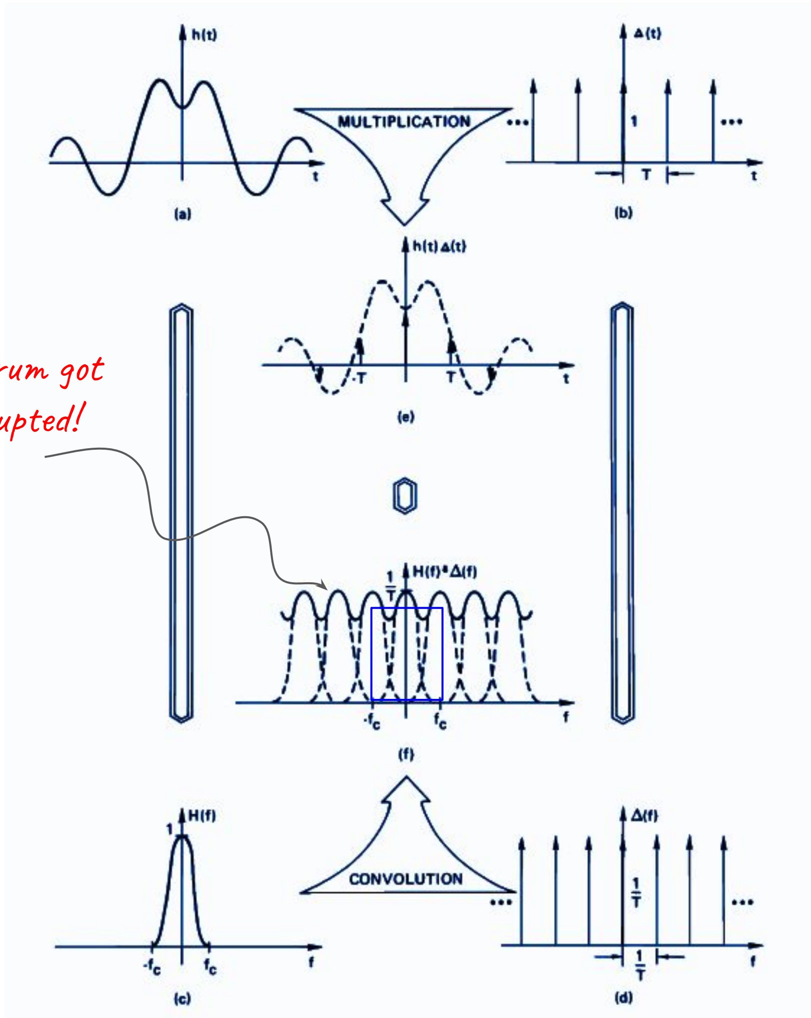
A. Did it reconstruct $y_1(t)$ or $y_2(t)$ or $y_3(t)$? Explain.
B. What is this phenomenon called?
C. How to avoid it?

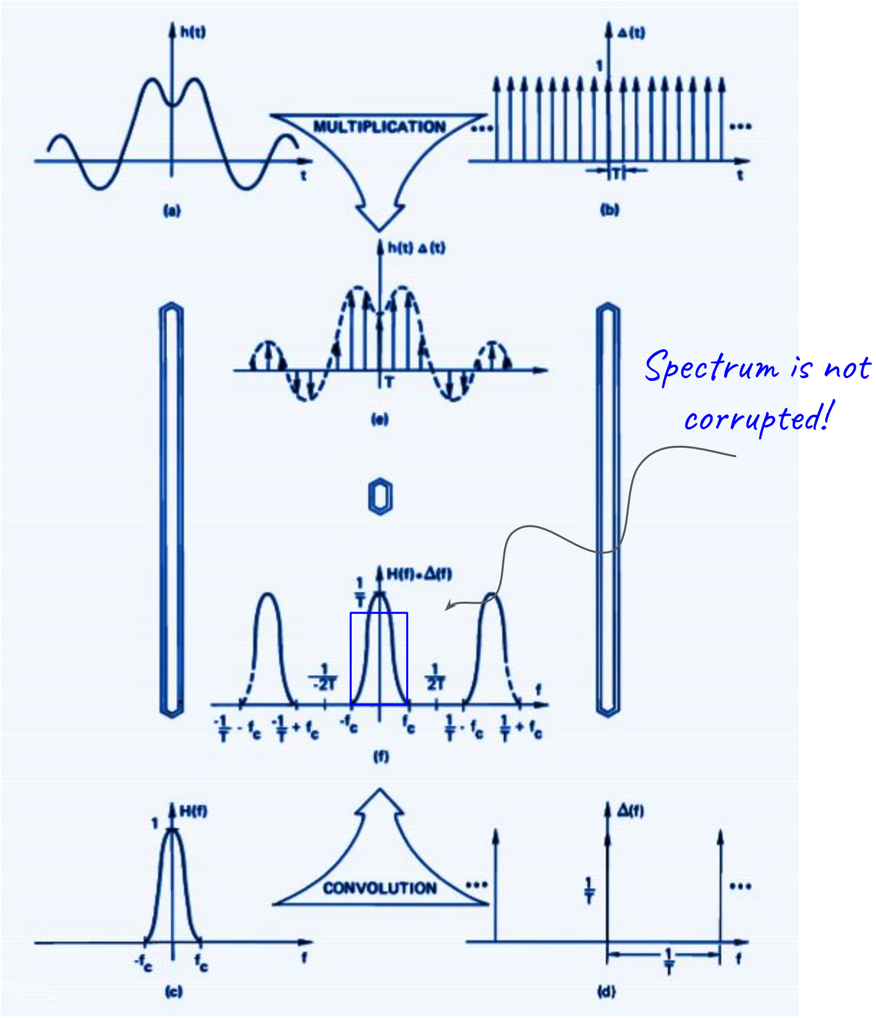
Aliasing

time-domain

frequency-domain

Spectrum got corrupted!



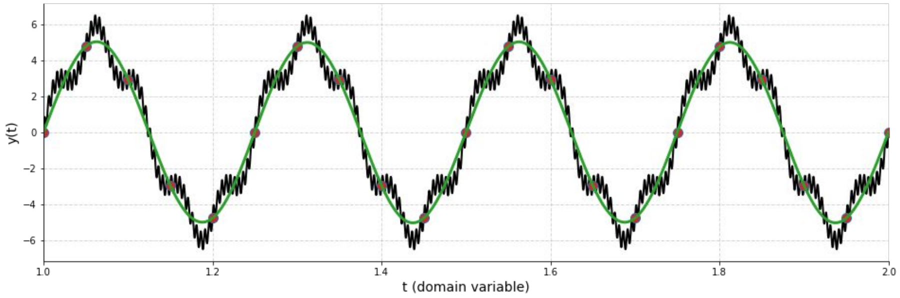


time-domain

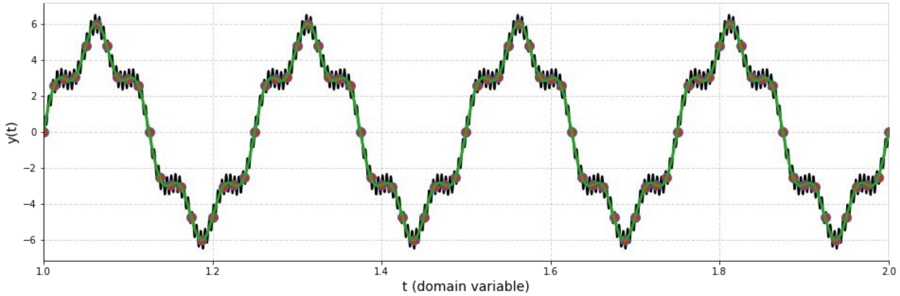
frequency-domain

Perfect reconstruction

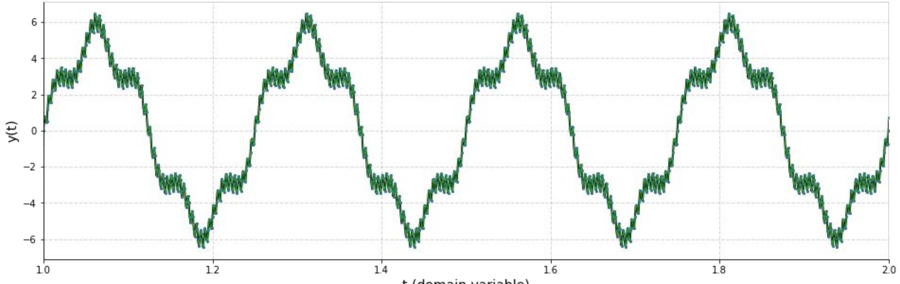
- $f_s = 20$ Hz



- $f_s = 80$ Hz



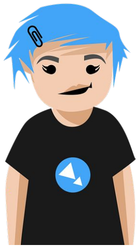
- $f_s = 1000$ Hz



For perfect reconstruction: obey Nyquist rate

$$f_s > 2\, f_{max}$$

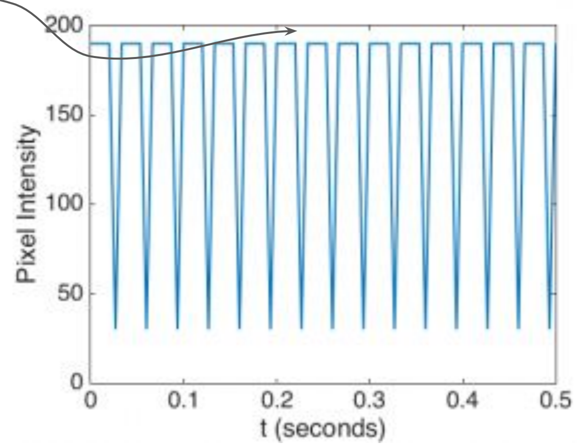
A Sampling Story



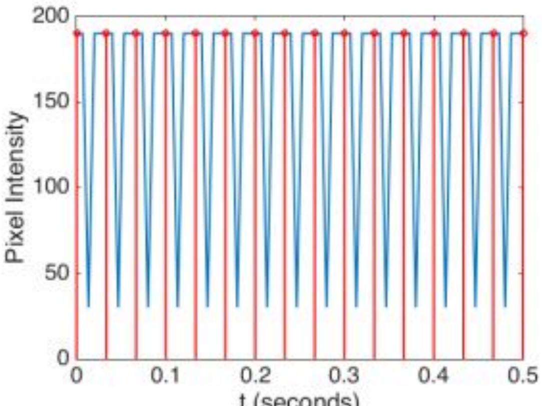
Moral of the story:

- Nature is continuous.
- Acquire domain knowledge.
- Check the sampling rate.

Value of intensity as a function of time on the camera pixel

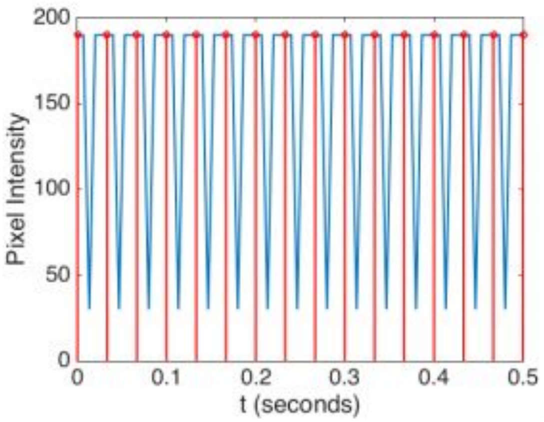


The period of this signal corresponds to the rotor speed.



Courtesy:
https://allsignalprocessing.com/2017/04/03/aliasing-movies-levitating-helicopters/

Value of intensity as a function of time on the camera pixel



The period of this signal corresponds to the rotor speed.

The camera samples this signal at a particular frame rate.

Summary

- Visualized sampling and interpolation
- Aliasing as folding of the spectrum
  - Results in corruption of the spectrum and hence of the reconstructed/interpolated signal
- Ways to avoid aliasing
  - Sample at higher and higher frequency
    - How do you decide? Physics? Experiments? Domain knowledge can help.
  - Use anti-aliasing filter
    - Don't capture the full spectrum but what whatever is captured is not corrupted
- Aliasing in real world - examples, video, images, sound and more