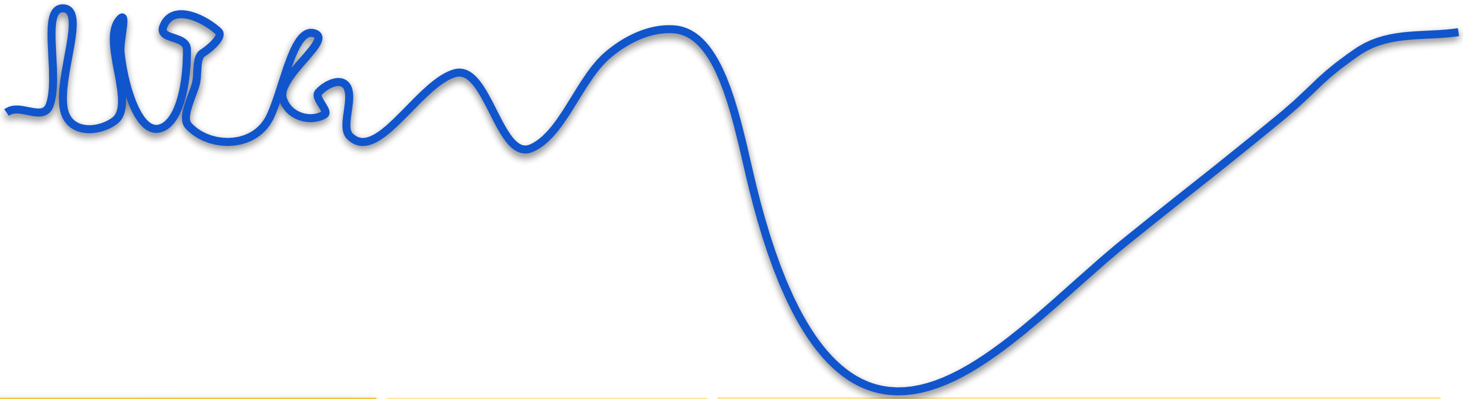
Computing with Signals



DA 623

Jan - May 2023

IIT Guwahati

Instructors: Neeraj Sharma

Lecture-20-[24-Feb]

Recap

Discrete Fourier Transform (DFT)

$$F(s_m) = \sum_{n=0}^{N-1} f(t_n) e^{-2\pi i s_m t_n}$$

$$= \sum_{n=0}^{N-1} f(t_n) e^{-2\pi i n m / 2BL}$$

$$= \sum_{n=0}^{N-1} f(t_n) e^{-2\pi i n m / N}$$

Assumptions:

- *f(t)* is (effectively) finite length (bandwidth) in time and frequency
- Duration (length): L
- Bandwidth = 2B (-B to +B)

Discrete Fourier Transform (DFT)

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$$\omega_N = e^{2\pi i/N}$$

$$\operatorname{Re}\omega_N = \cos 2\pi/N\,, \quad \operatorname{Im}\omega_N = \sin 2\pi/N$$

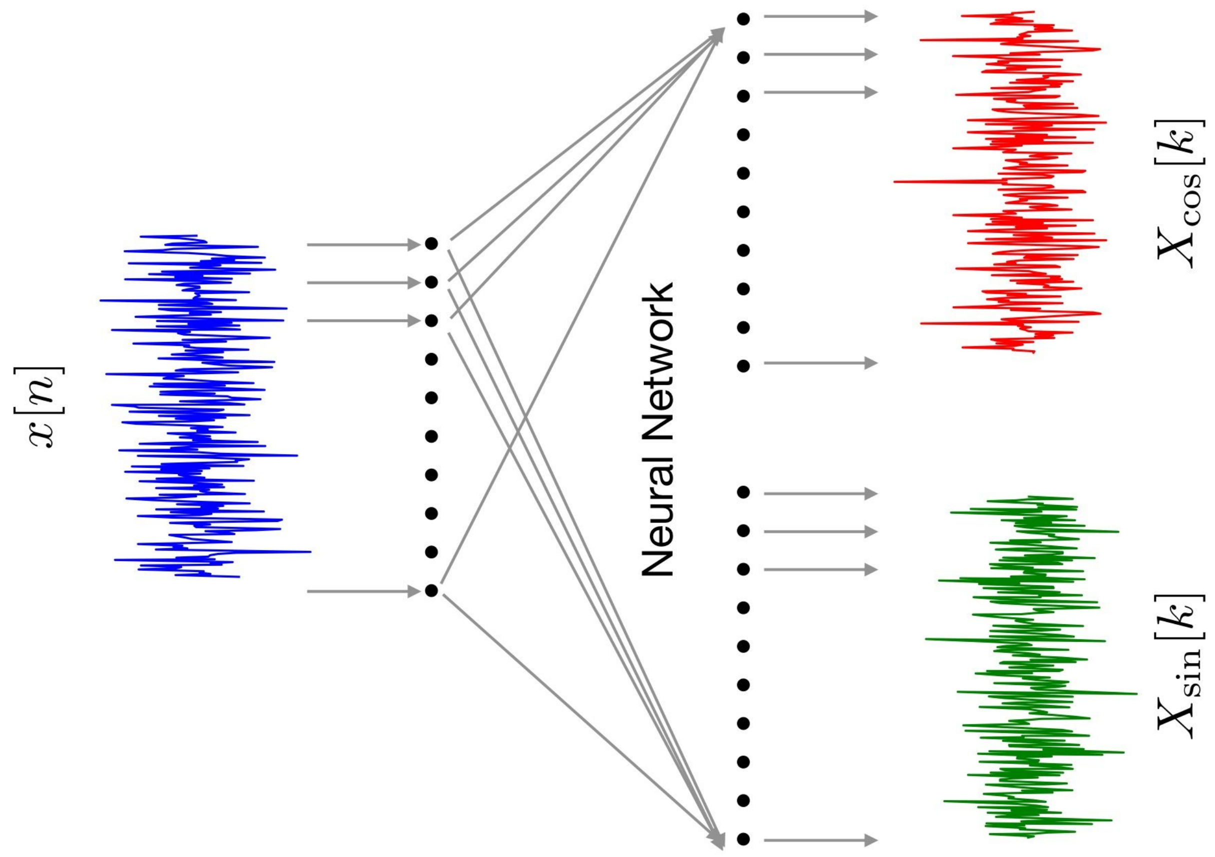
Connecting with the concept of roots of unity - lying on a circle in the complex plane

$$X_k = \frac{1}{N} \sum_{n=0}^{N-1} x_n e^{i 2\pi k \frac{n}{N}}$$

To find the energy at a particular frequency, spin your signal around a circle at that frequency, and average a bunch of points along that path.

Courtesy: https://betterexplained.com/

Switch gears - Can we train a Neural Network to output Fourier Transform?



$$\mathrm{DFT}(\mathrm{x[n]}) = X_{\cos}[k] + jX_{\sin}[k]$$

How to train such as network?

- What should be the training/validation/test data
- In what form should we give the input
- Should we have any non-linearity in the network
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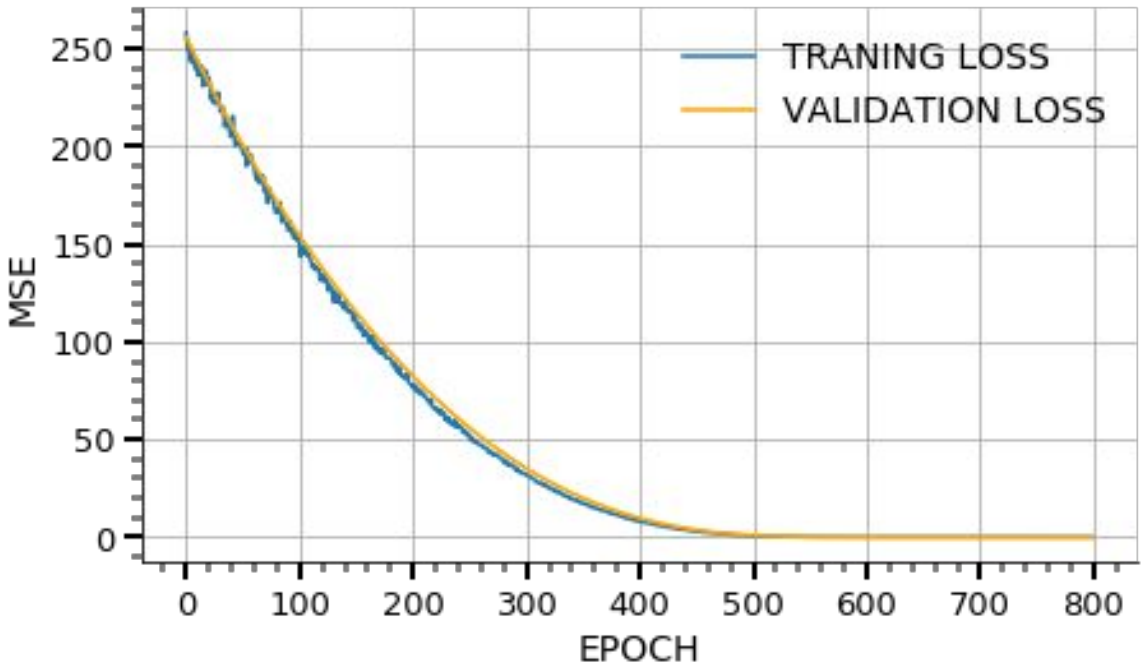
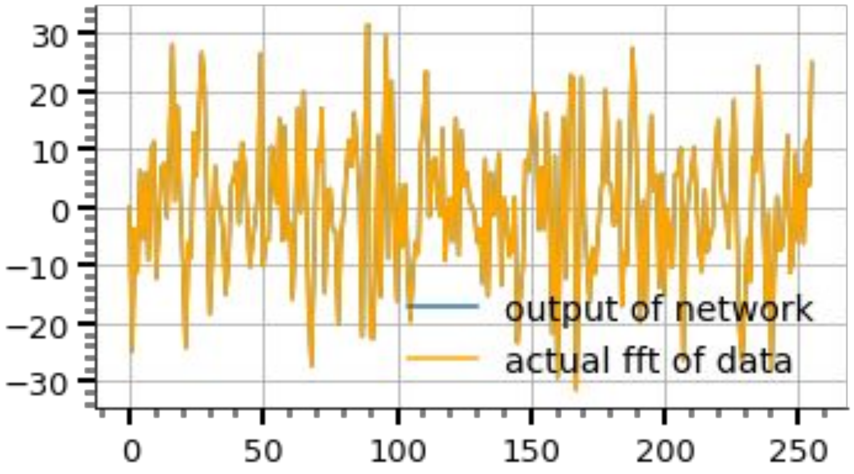
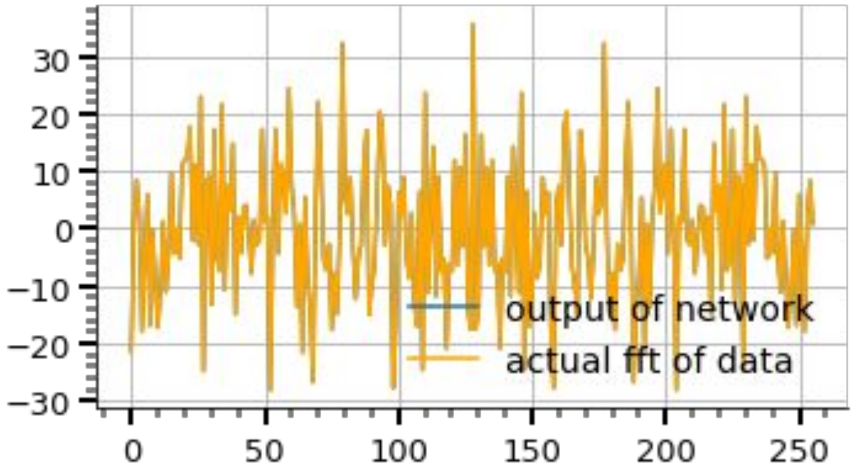
Discussion on board

Epoch-wise loss

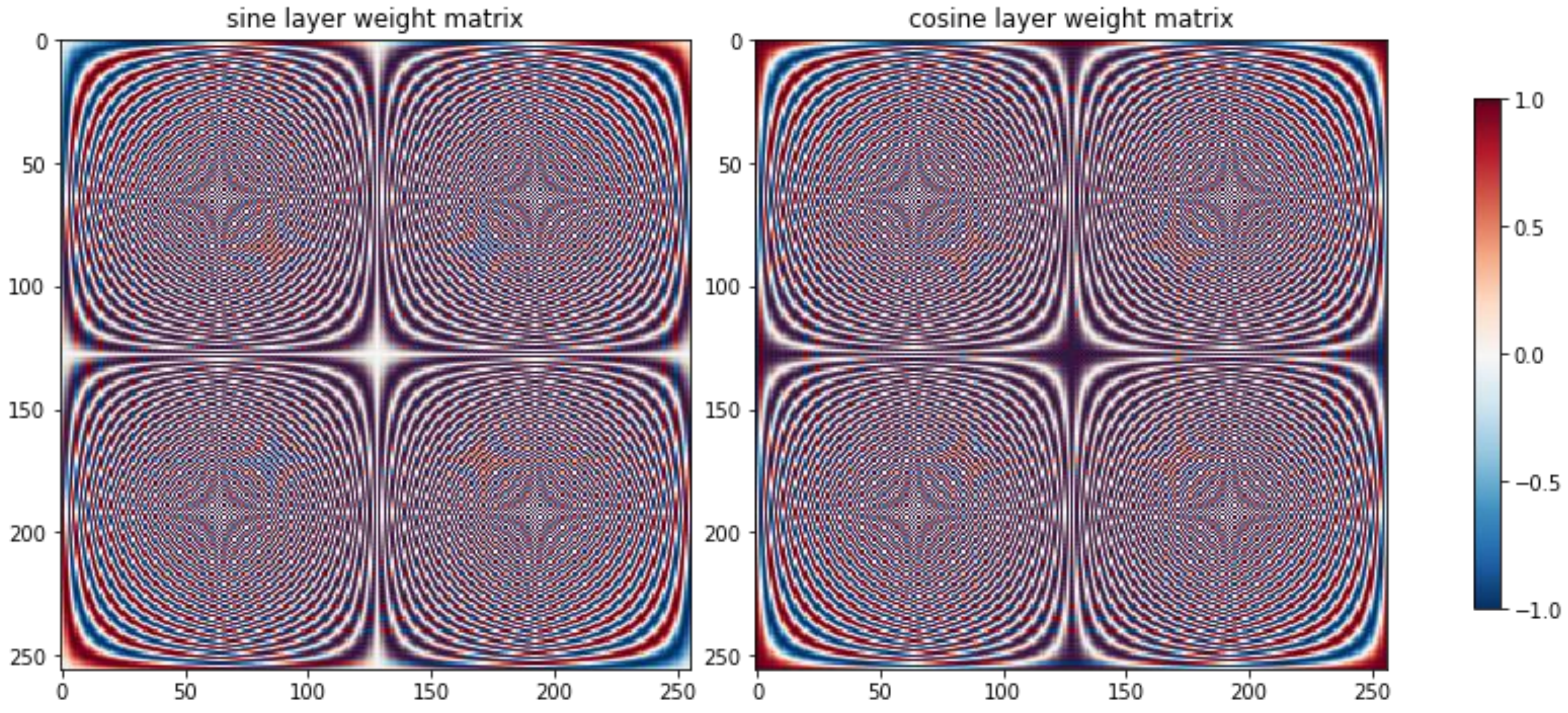


How good is the estimation?

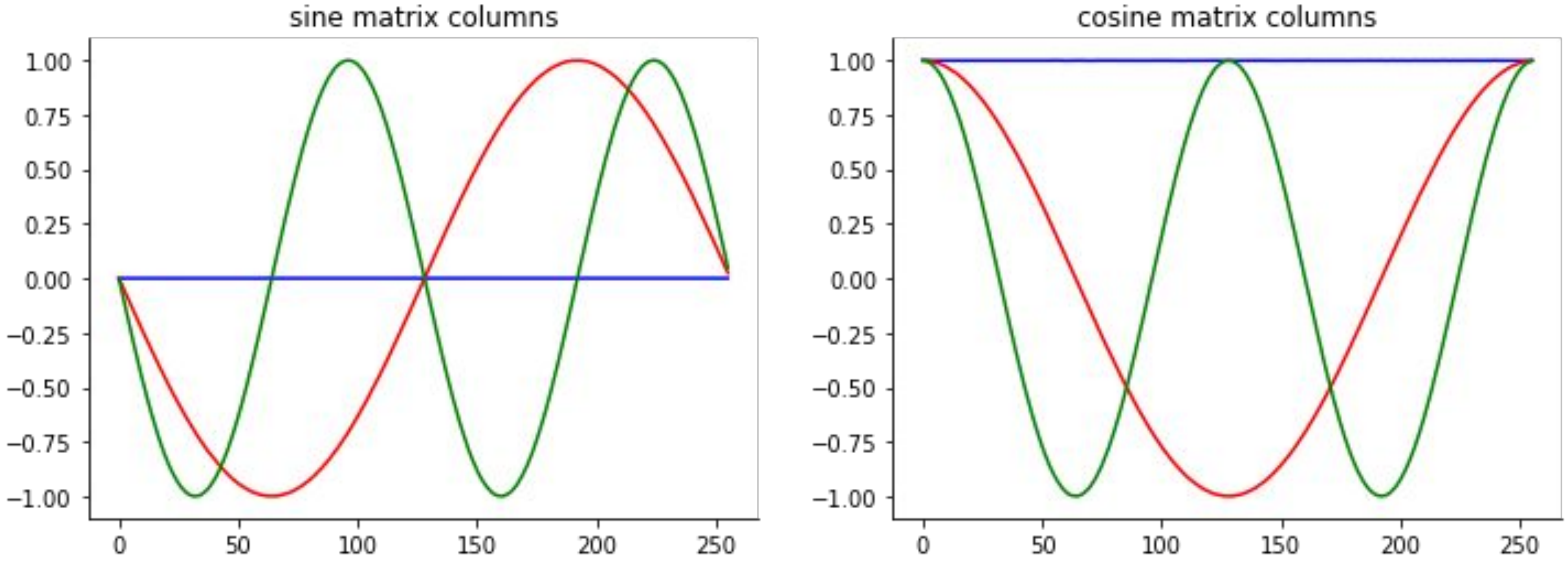




Visualizing the NN weights



It learnt the sine and cosine basis!



Thank you!