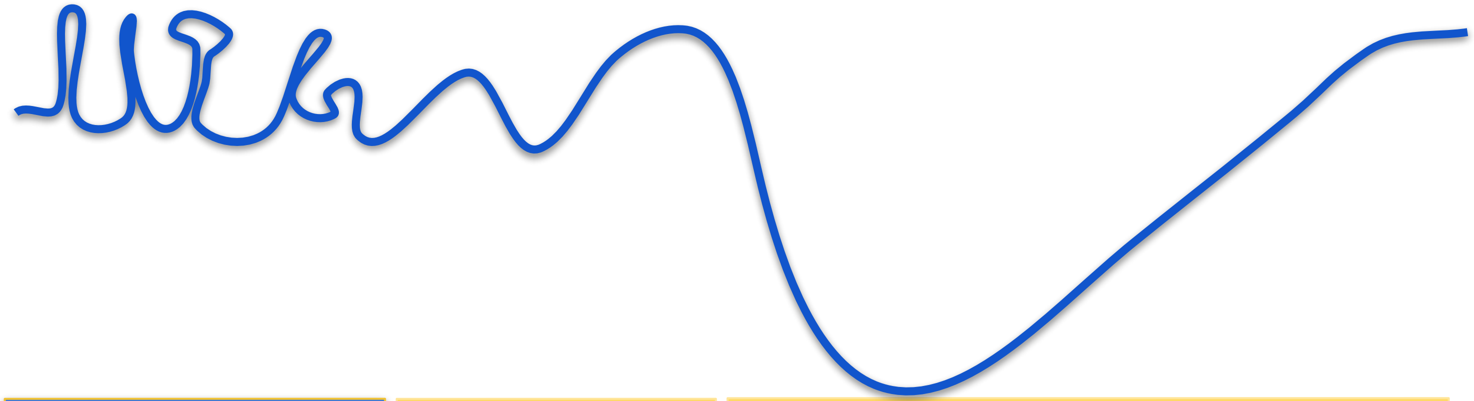
Computing with Signals



DA 623

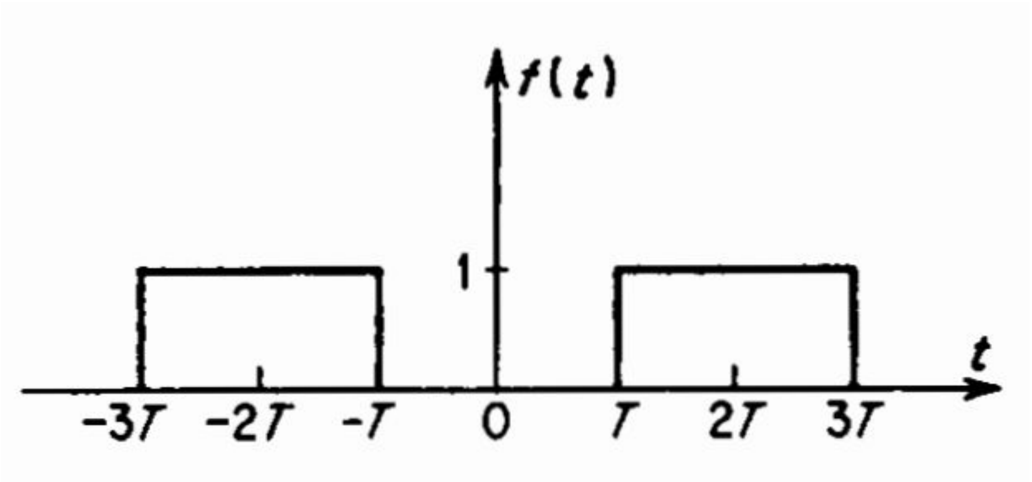
Jan - May 2023

IIT Guwahati

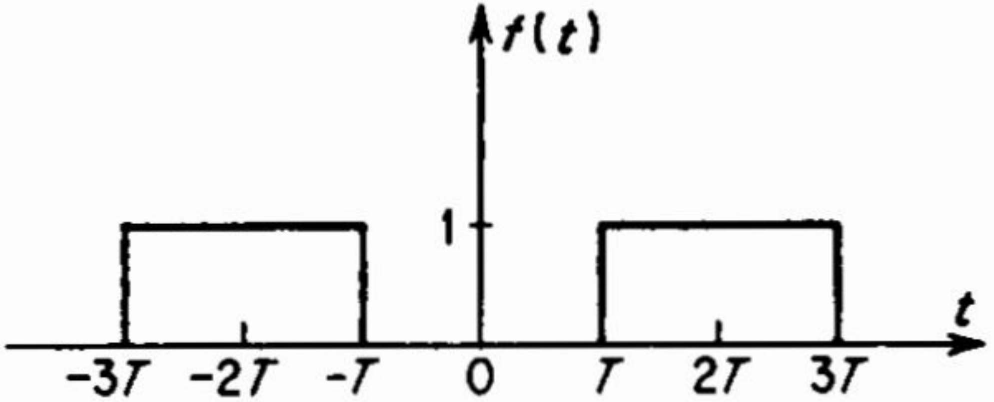
Instructors: Neeraj Sharma

Lecture-28 [6th Apr]

Review Question



What will the Fourier transform of f(t)?



What will the Fourier transform of f(t)?

$$F(\omega) = \frac{2\sin\omega T}{\omega}\left(e^{-j2T\omega} + e^{j2T\omega}\right) = \frac{4\sin\omega T}{\omega}\cos 2\omega T$$

Recap:
Prev Lecture

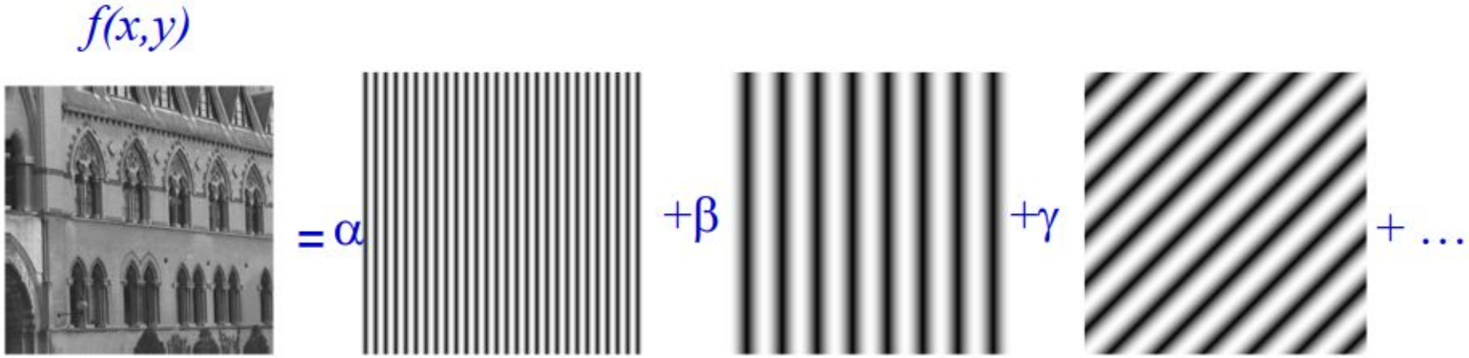
2-D Fourier Transform

Summary

The spatial function $f(x, y)$

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v) e^{j2\pi(ux+vy)}\, du\, dv$$

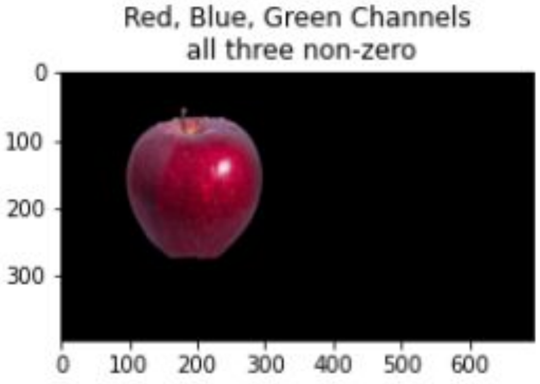
is decomposed into a weighted sum of 2D orthogonal basis functions in a similar manner to decomposing a vector onto a basis using scalar products.



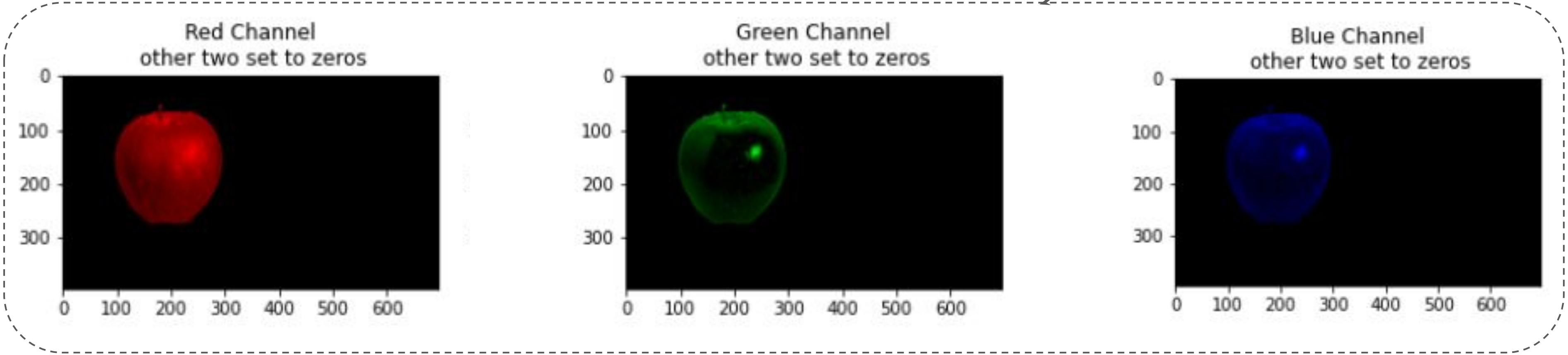
What to do for color images?

Treat each channel (R, G, B) independently

Do 2-D Fourier transform on each channel.



Remember: a color image has 3 channels
(R, G, B)?



This lecture
Switch Gears

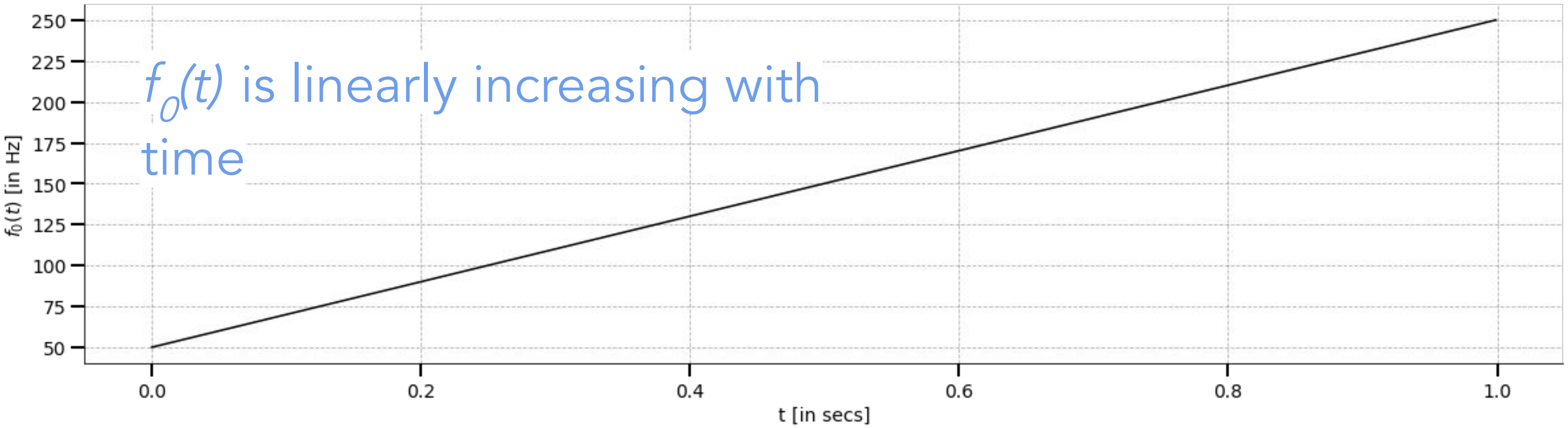
Time-Frequency Analysis

[Bringing time into frequency space]

- In previous lectures, we assumed frequency is constant.
- Example, $x(t) = a\sin(2\pi f_0 t + \phi)$
- Can we have a signal model in which frequency can vary with time?

What do we mean by frequency varies with time? f_0 is now $f_0(t)$.

Example:



Generalizing the sine wave signal to include frequency variation

Time-invariant sine wave signal model

$$x(t) = a \sin \phi(t)$$

$$\phi(t) = 2\pi \int_{-\infty}^{t} f_0 d\tau = 2\pi f_0 t$$

$$x(t) = a \sin 2\pi f_0 t$$

Amplitude (a) is constant

Frequency (f_0) is a constant

Only phase is varying with time and only in a linear way.

Time-varying sine wave signal model

$$x(t) = a(t) \sin \phi(t)$$

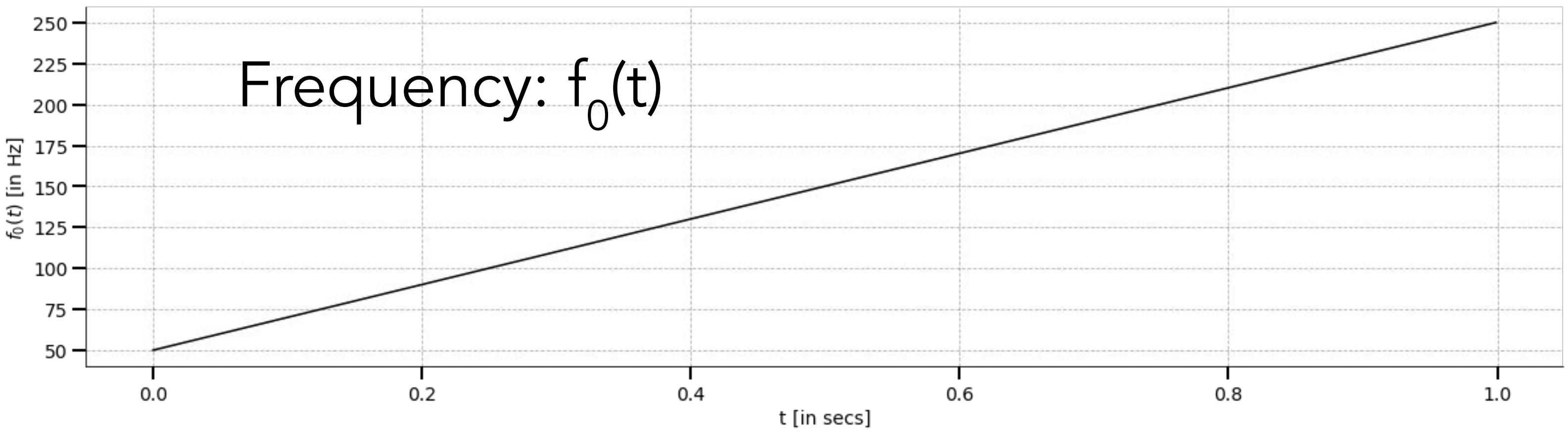
$$\phi(t) = 2\pi \int_{-\infty}^{t} f_0(\tau) d\tau$$

Amplitude: $a(t)$

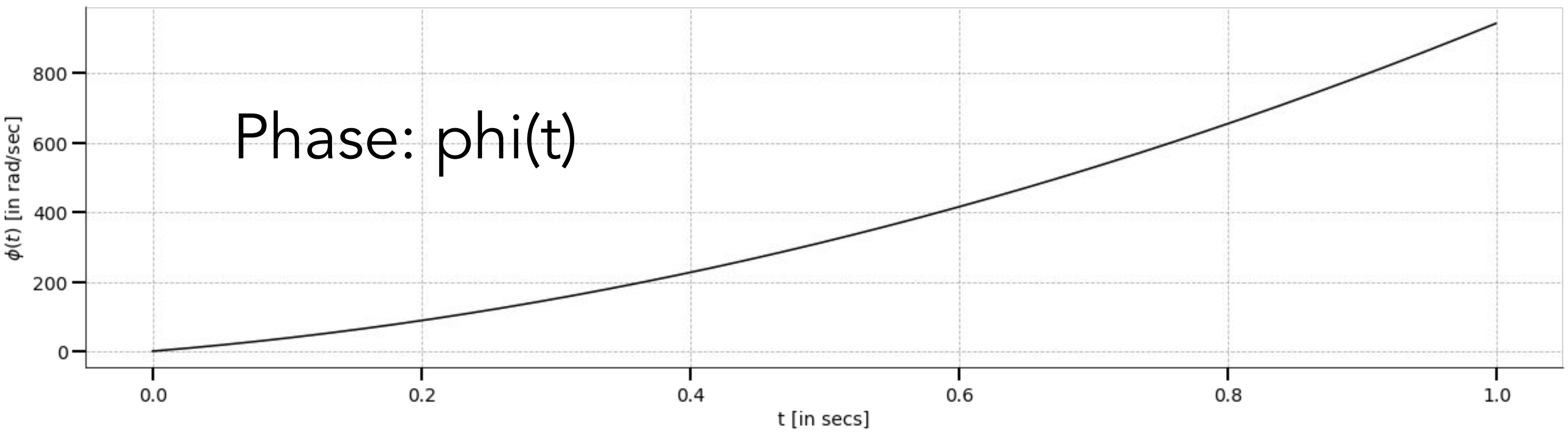
Frequency: $f_0(t)$

Phase: $\phi(t)$

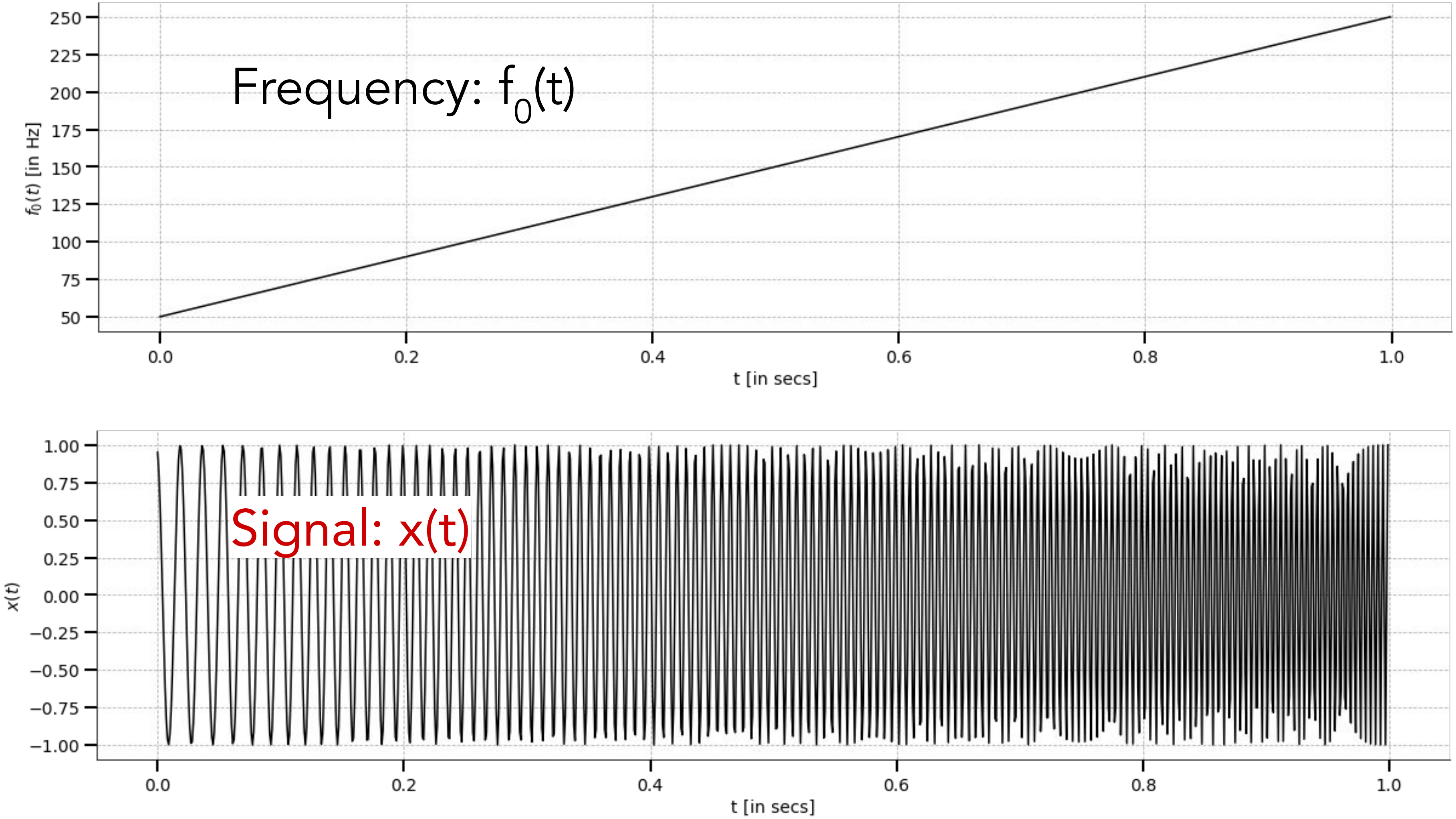
All three are varying with time.
Phase variation can be non-linear; dependent on frequency variation



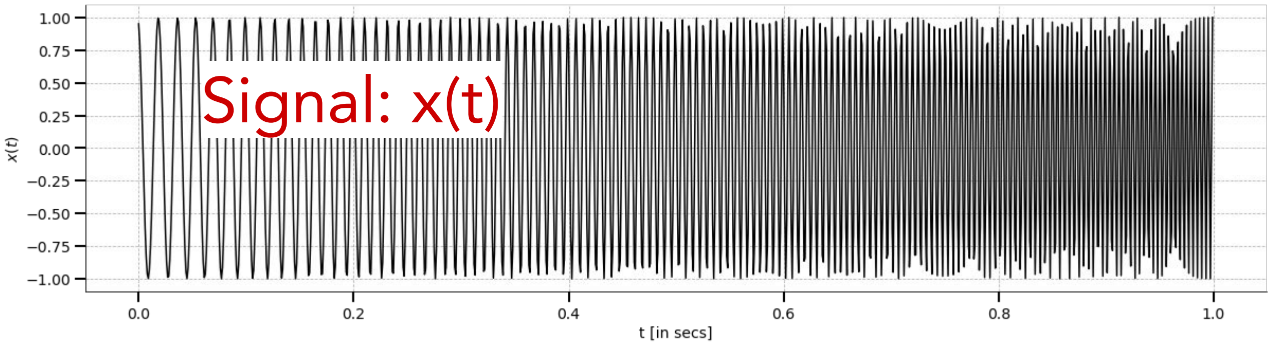
Frequency: $f_0(t)$
250
225
200
175
150
125
100
75
50
$f_0(t)$ [in Hz]
0.0
0.2
0.4
0.6
0.8
1.0
t [in secs]



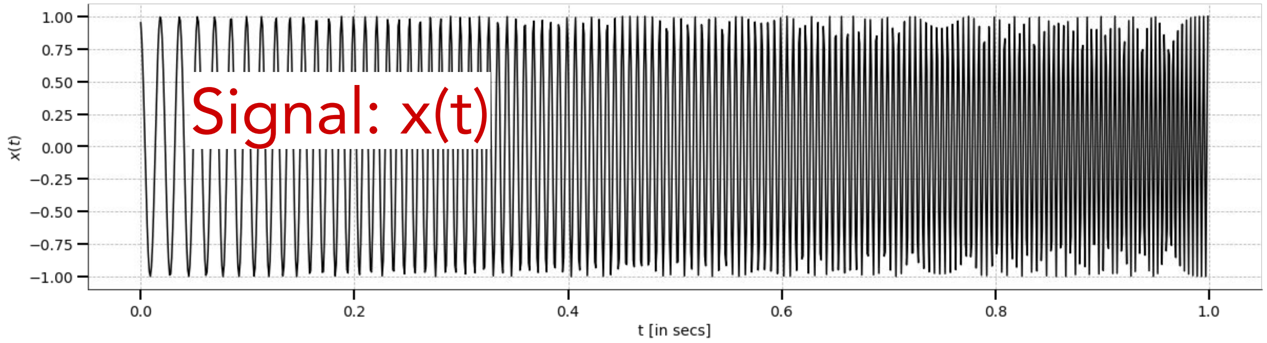
Phase: phi(t)
800
600
400
200
0
$\phi(t)$ [in rad/sec]
0.0
0.2
0.4
0.6
0.8
1.0
t [in secs]



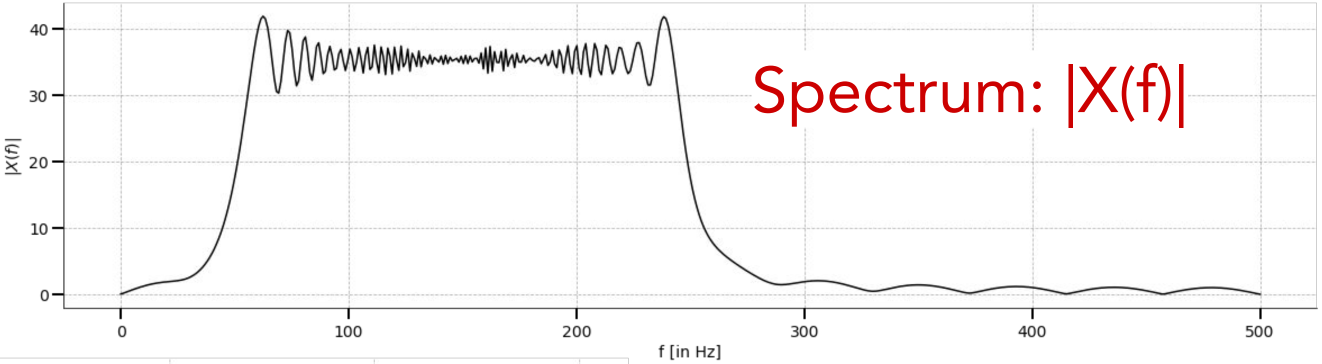
Frequency: $f_0(t)$
$f_0(t)$ [in Hz]
t [in secs]
Signal: x(t)
x(t)
t [in secs]



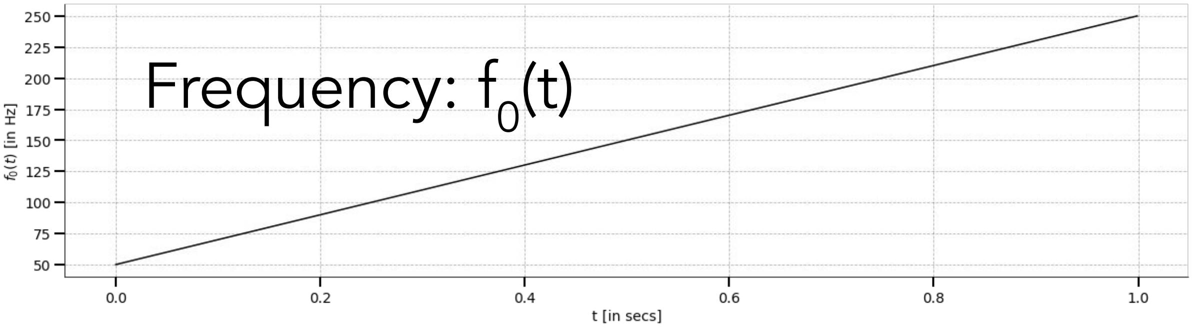
Signal: x(t)
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
x(t)
0.0
0.2
0.4
0.6
0.8
1.0
t [in secs]



Signal: x(t)
x(t)
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
0.2
0.4
0.6
0.8
1.0
t [in secs]

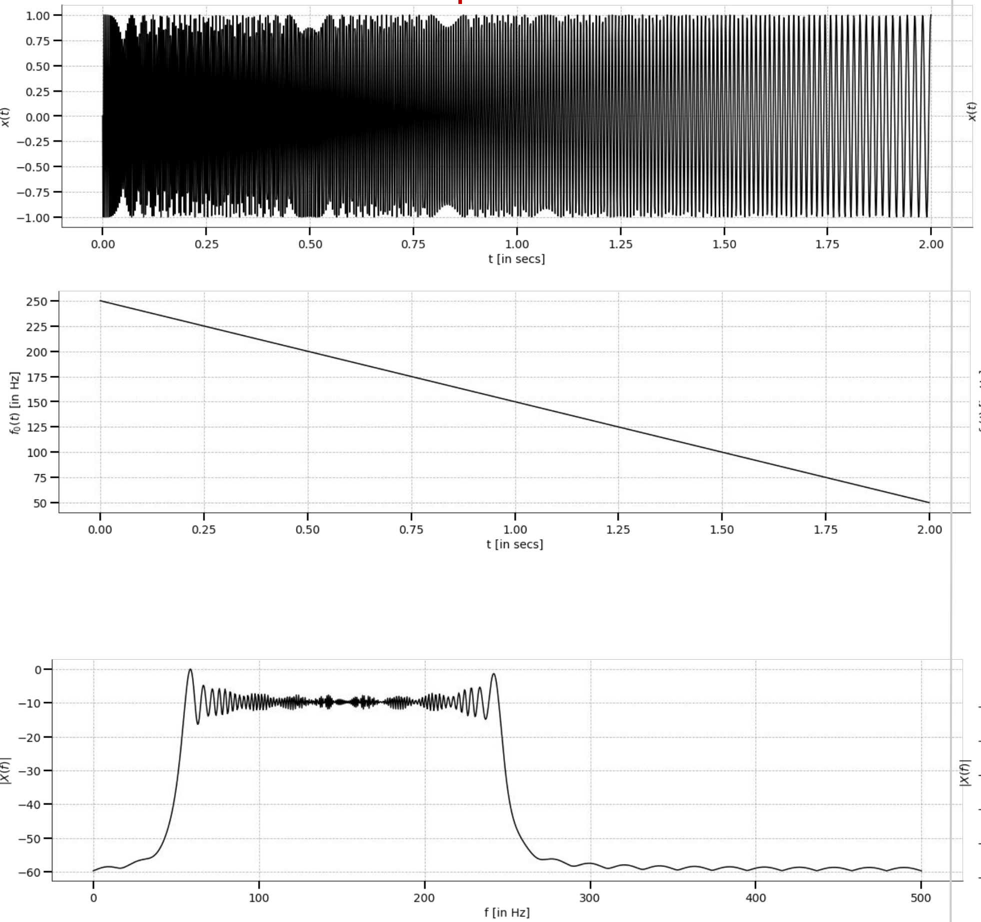


Spectrum: |X(f)|
|X(f)|
40
30
20
10
0
0
100
200
300
400
500
f [in Hz]

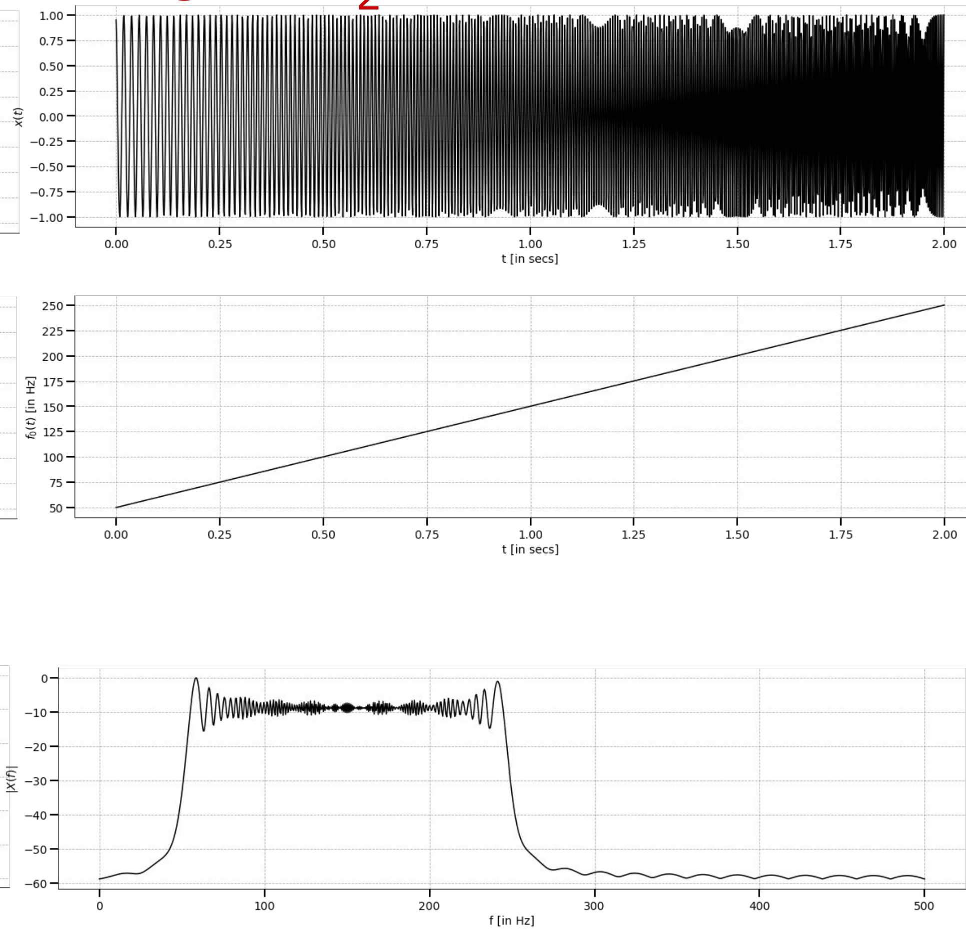


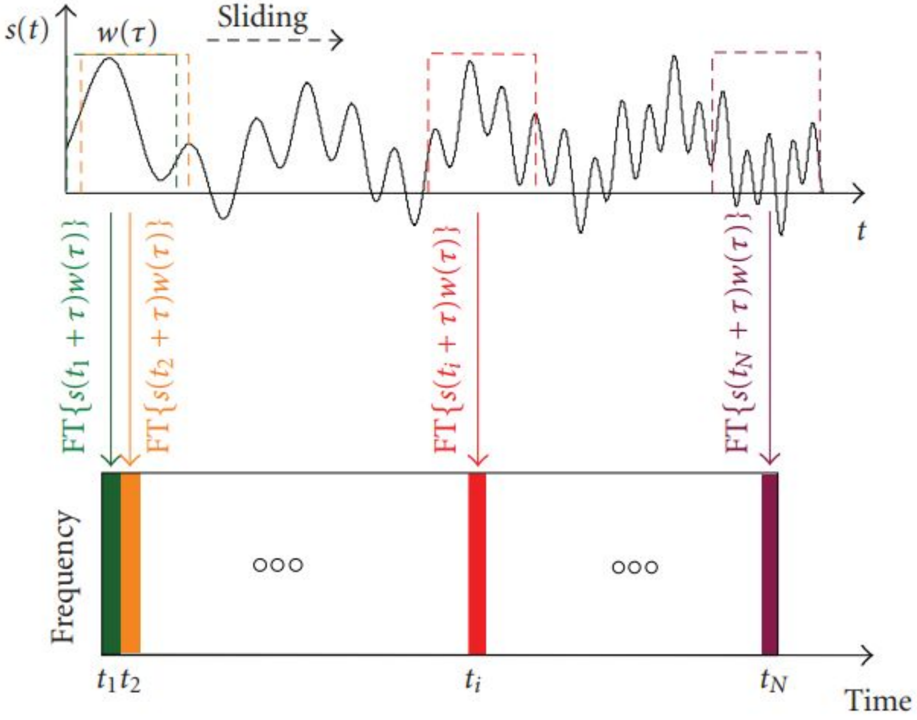
Frequency: f0(t)
f0(t) [in Hz]
250
225
200
175
150
125
100
75
50
0.0
0.2
0.4
0.6
0.8
1.0
t [in secs]

Signal: $x_1(t)$



Signal: $x_2(t)$

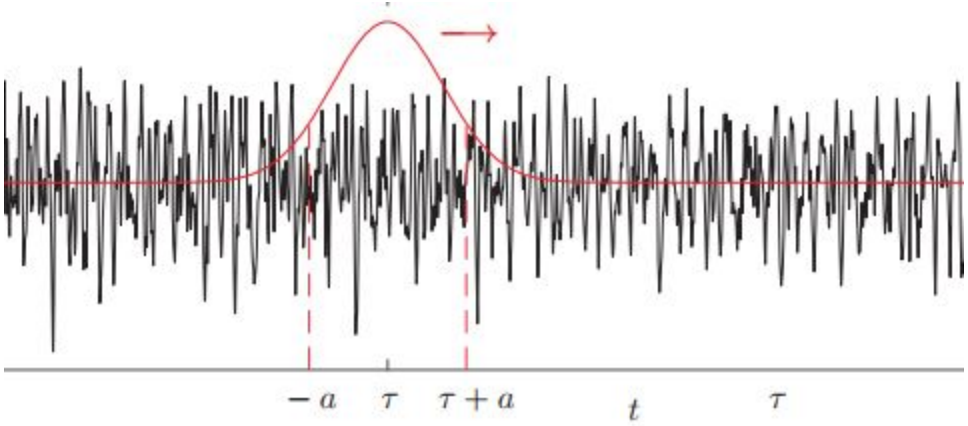


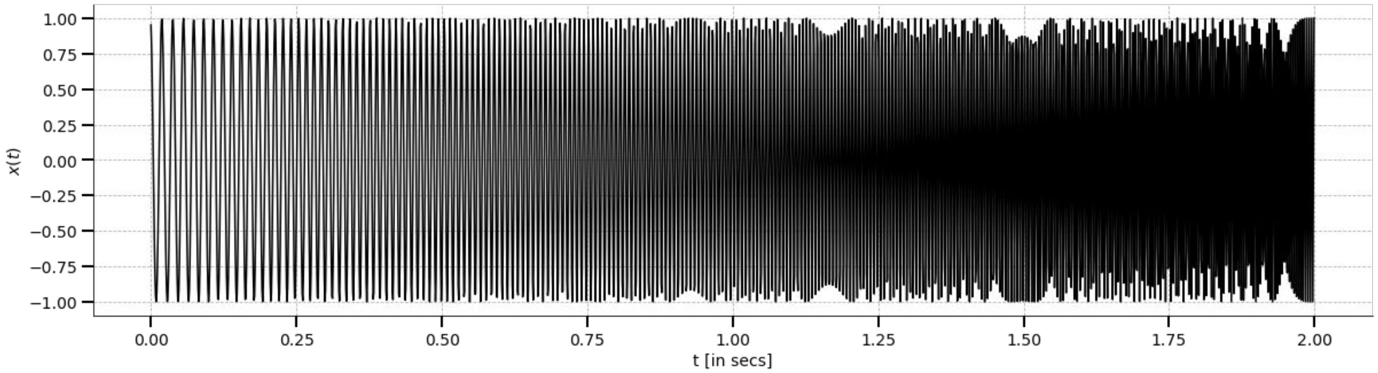


Srdjan Stankovic, Time-Frequency Analysis and Its Application in Digital Watermarking, Journal EURASIP, 2010

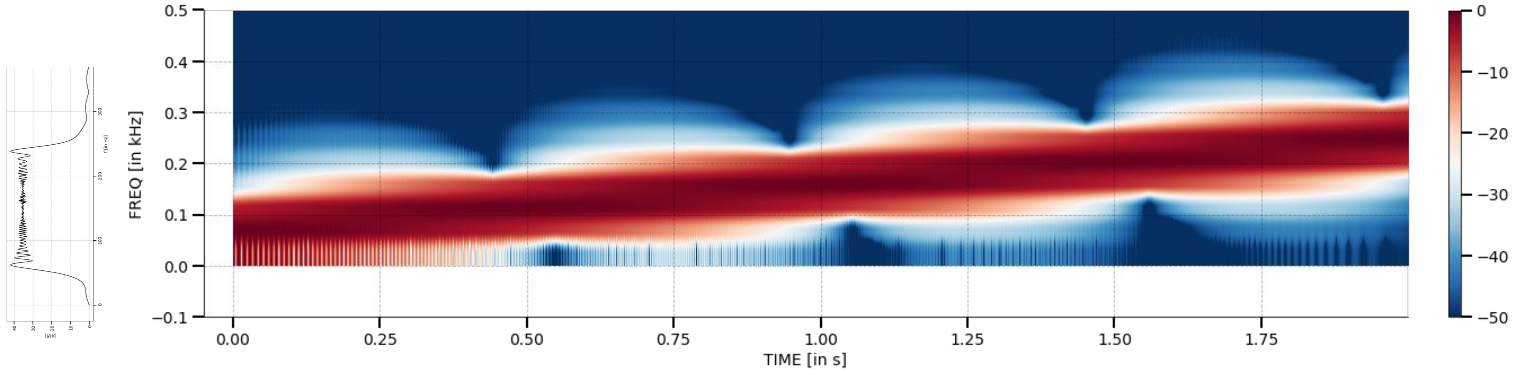
Short-time Fourier Transform (STFT)

$$X(t,f) = \int_{-\infty}^{\infty} w(t-\tau)x(\tau)e^{-j2\pi f\tau}\,d\tau$$





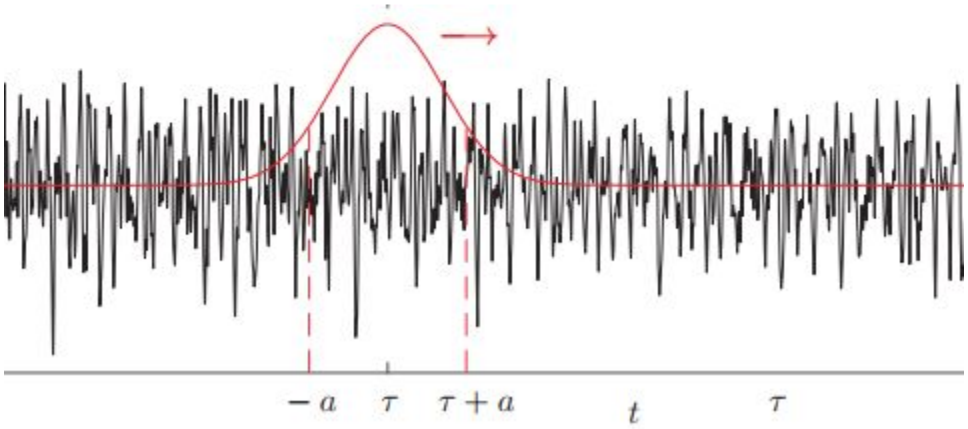
x(t)
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
t [in secs]



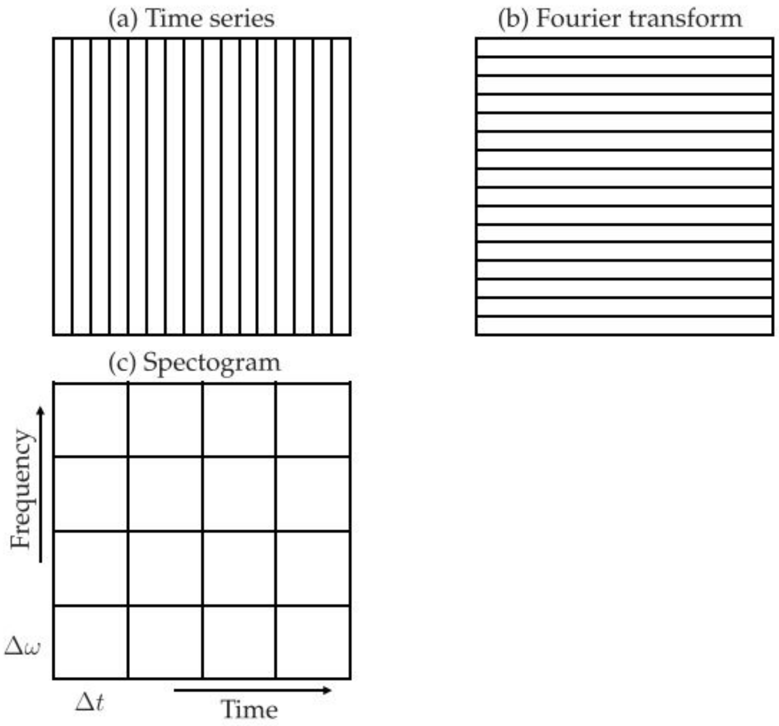
FREQ [in kHz]
0.5
0.4
0.3
0.2
0.1
0.0
−0.1
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
TIME [in s]
0
−10
−20
−30
−40
−50

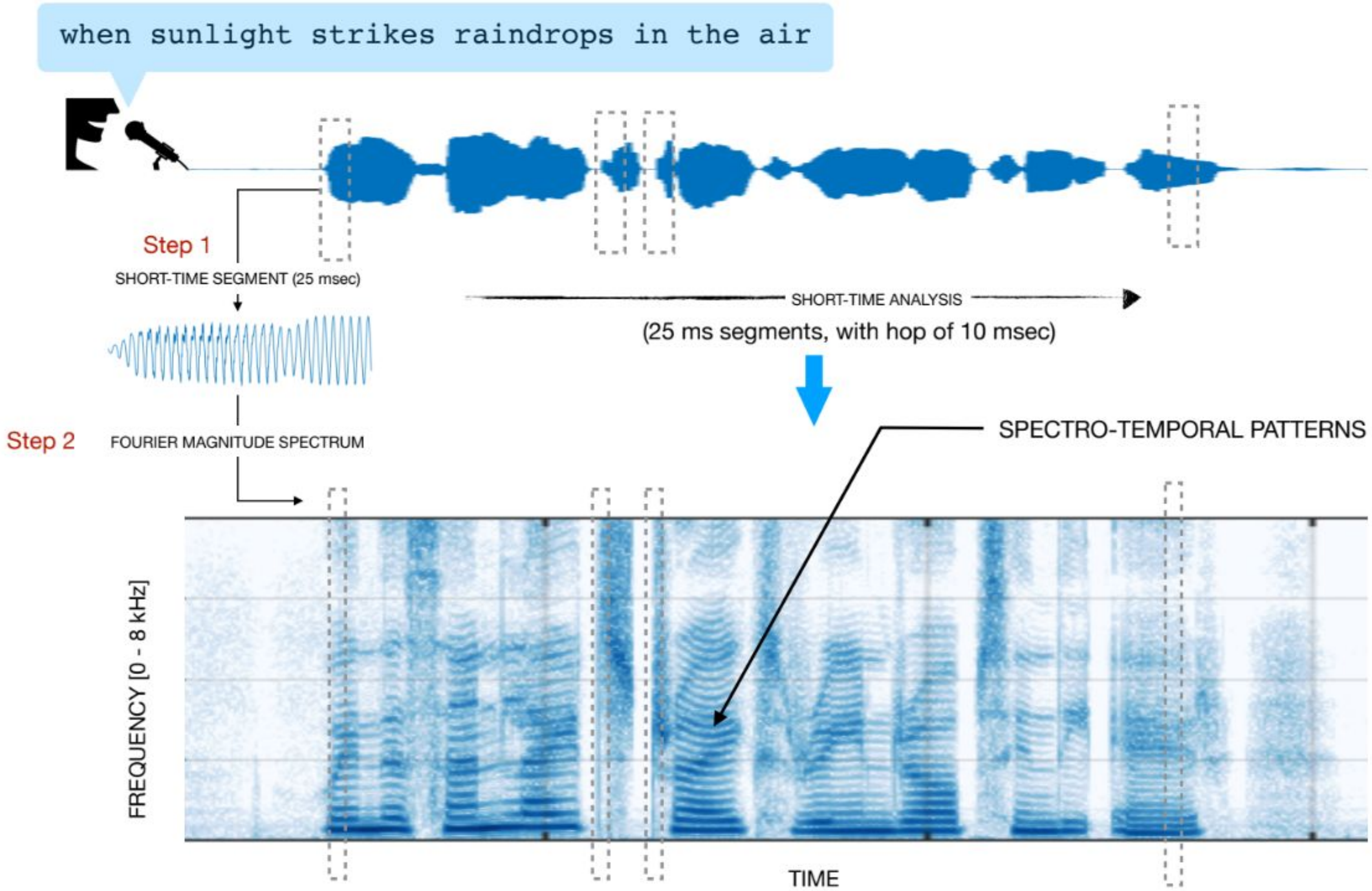
Short-time Fourier Transform (STFT)

$$X(t, f) = \int_{-\infty}^{\infty} w(t - \tau)x(\tau)e^{-j2\pi f\tau}\,d\tau$$



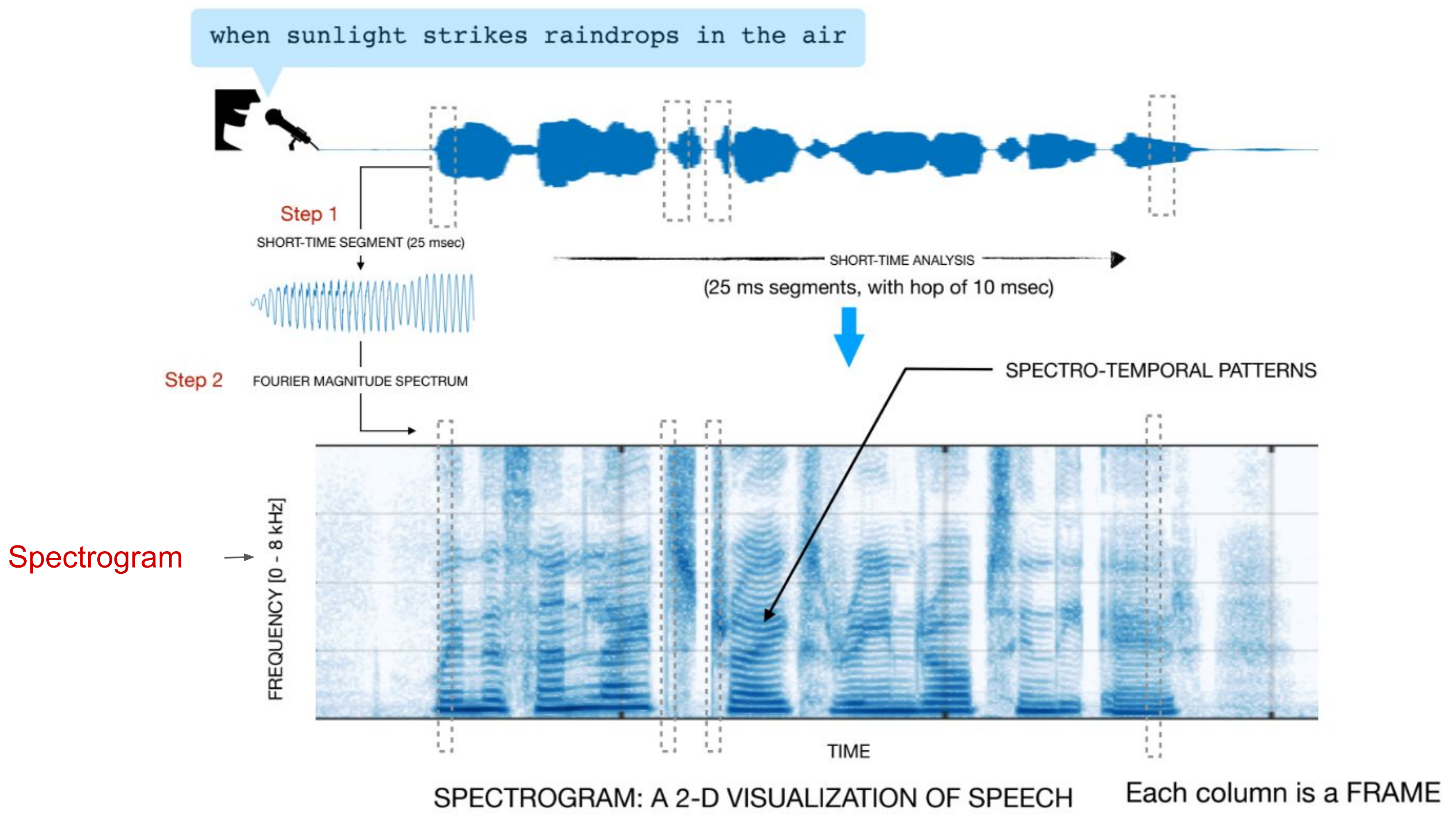
Resolution





SPECTROGRAM: A 2-D VISUALIZATION OF SPEECH

Each column is a FRAME


when sunlight strikes raindrops in the air
Step 1
SHORT-TIME SEGMENT (25 msec)
SHORT-TIME ANALYSIS
(25 ms segments, with hop of 10 msec)
Step 2
FOURIER MAGNITUDE SPECTRUM
SPECTRO-TEMPORAL PATTERNS
Spectrogram
FREQUENCY [0 - 8 kHz]
TIME
SPECTROGRAM: A 2-D VISUALIZATION OF SPEECH
Each column is a FRAME

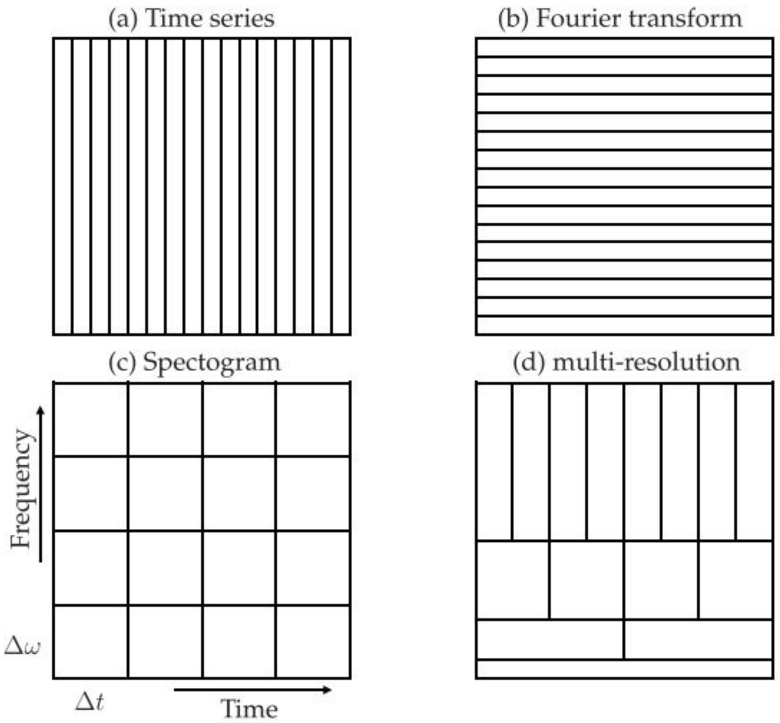
Multi-resolution analysis

- Wavelets

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)$$

$$\mathcal{W}_\psi(f)(a,b) = \langle f, \psi_{a,b}\rangle = \int_{-\infty}^{\infty} f(t)\bar{\psi}_{a,b}(t)\,dt$$

Resolution



Reading Material

Estimating and Interpreting The Instantaneous Frequency of a Signal—Part 1: Fundamentals

BOUALEM BOASHASH, SENIOR MEMBER, IEEE

THEORY OF COMMUNICATION*

By D. GABOR, Dr. Ing., Associate Member.†

Thank you